# Infectious Disease Management

*Insights from simple models*

# The Anatomy of an Epidemic

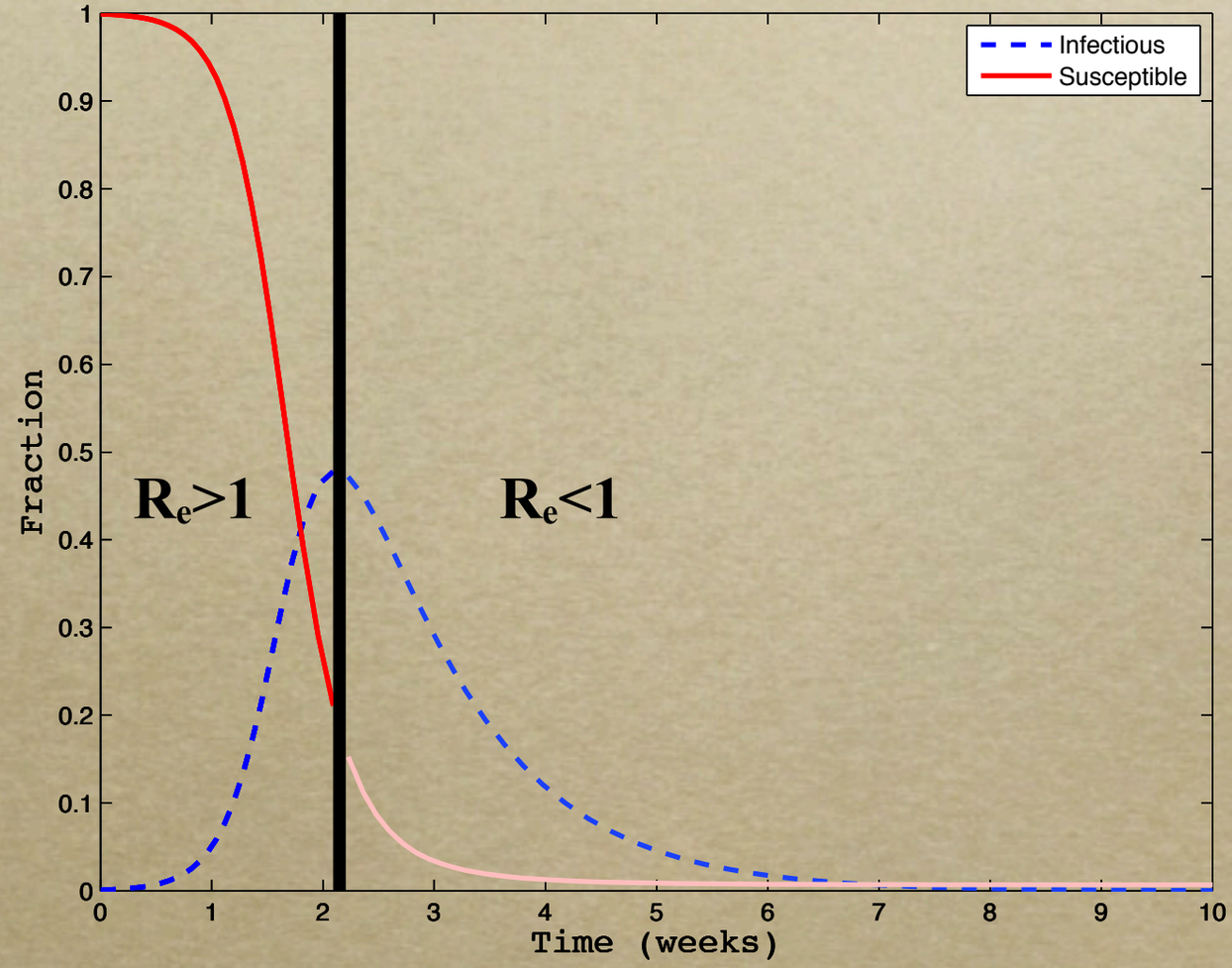
Initially, exponential growth (proportional to $R_0$)

But, depletes susceptibles, so $R_0$ no longer useful

Instead, define effective value of $R_0$ (call it $R_e$)

$R_e$ scales with proportion of susceptibles in population (s=X/N), ie $R_e = R_0 s$

*when $R_e$<1, each infectious individual infects fewer than one new person, breaking transmission chain*



# Vaccination

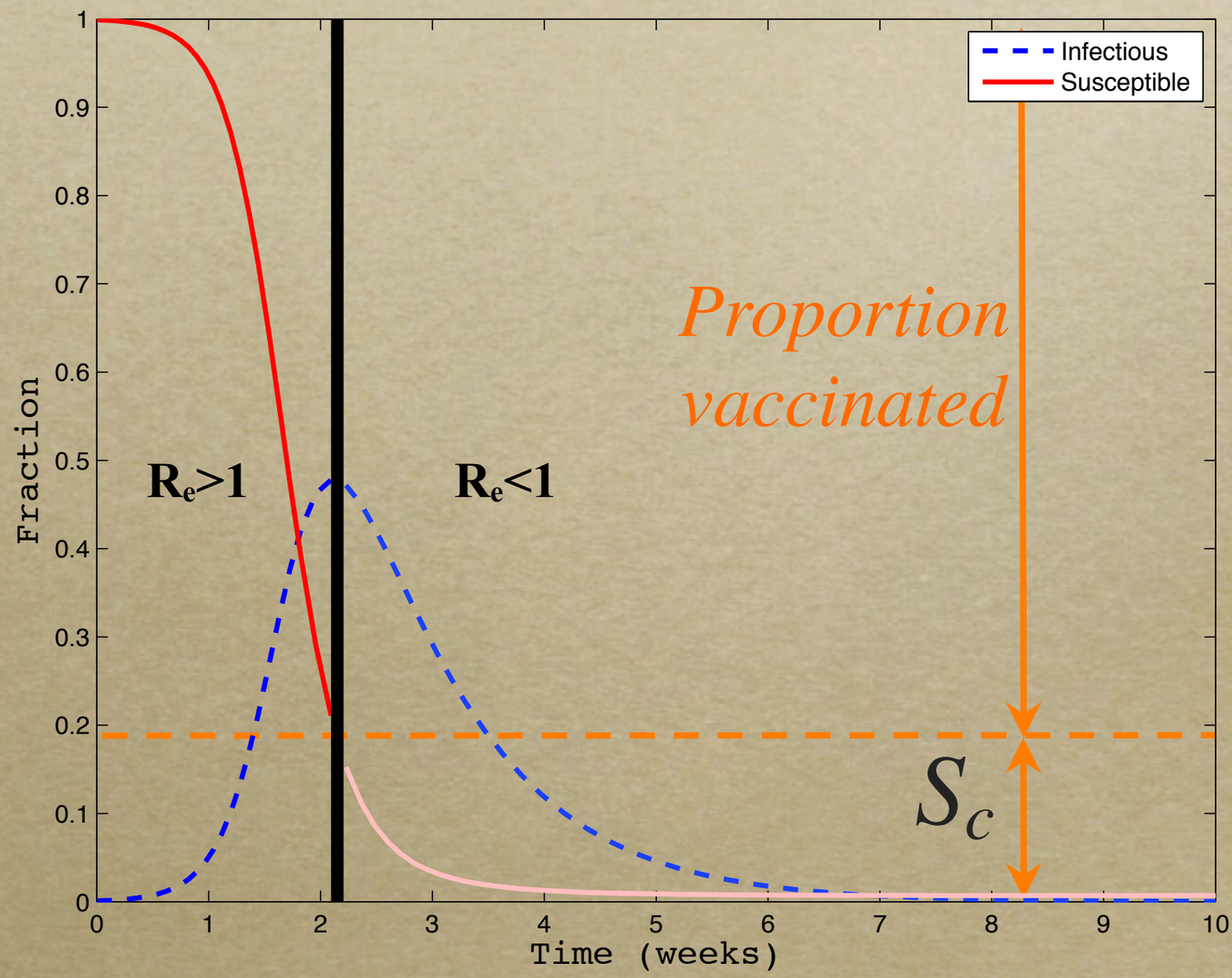
*If, by vaccination, we can reduce proportion of susceptibles below a <u>critical level, $S_c$,</u> then $R_e<1$ and infection can never 'invade'*

Recall: $R_{e_e} = R_0X/N$

So, $S_c=1/R_0$ represents $R_{e_e}=1$ and will achieve our goal

So, critical vaccination proportion to eradicate is

$$p_c = 1-S_c = 1-1/R_0$$



# Mathematically ...

- Consider rate of change of invectives:

$$\frac{dY}{dt} = \beta X \frac{Y}{N} - \gamma Y$$

- Hence, preventing initial spread (dY/dt <0) requires

$$\beta \frac{X}{N} < \gamma$$

$$\implies \frac{X}{N} < \frac{\gamma}{\beta} = \frac{1}{R_0}$$

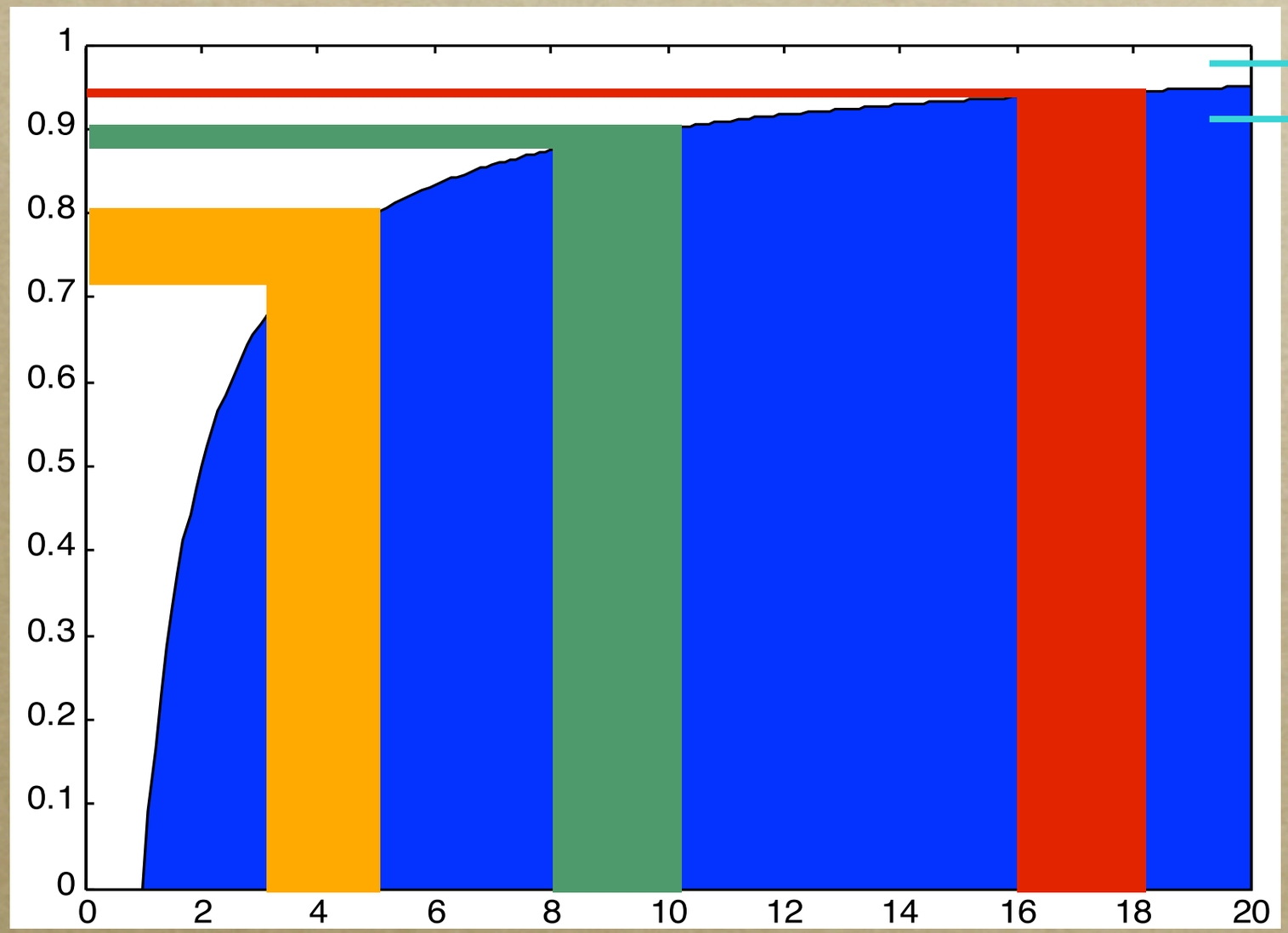
# Eradication Criterion

$$p_c = 1 - \frac{1}{R_0}$$

~~Smallpox~~

Polio

Measles



**Herd immunity:**

protection of an individual from infection via others in population gaining immunity

If neighbors have been vaccinated, probability of acquiring disease is lower

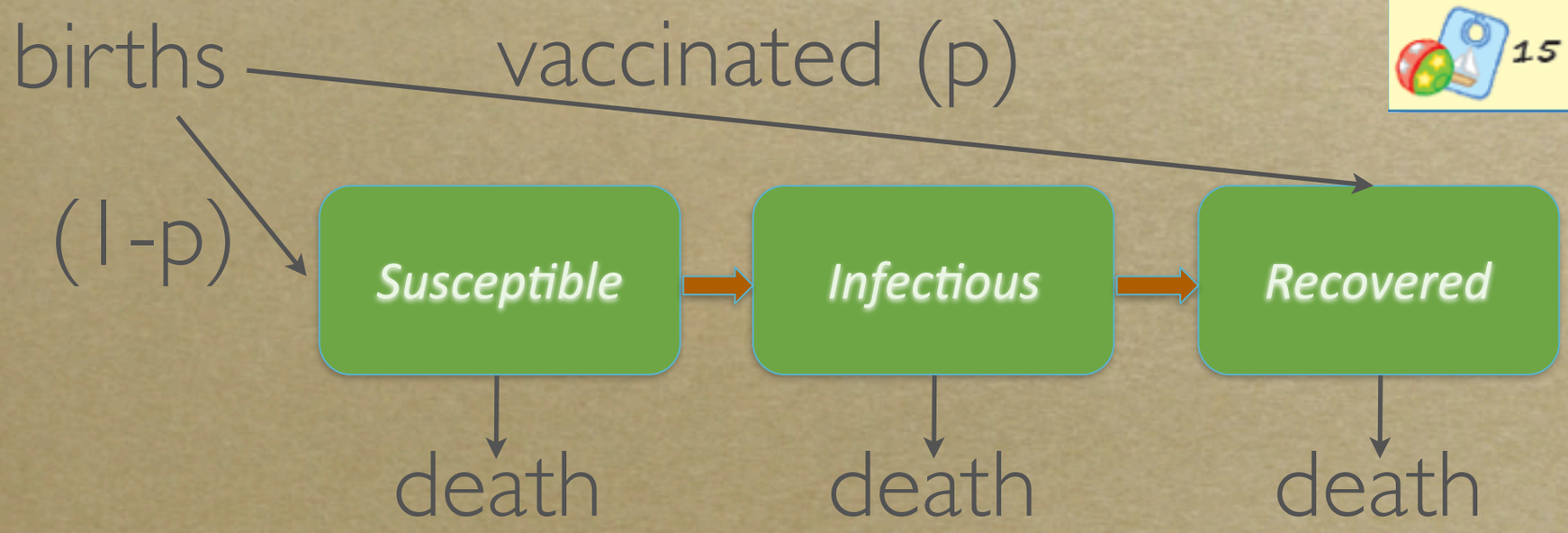
Don’t need to vaccinate everyone to eradicate an infectious disease

Extent of vaccination effort determined by simple quantity, $R_0$

# 1. “Paediatric immunization”

- *Familiar with infant immunization*
- *Generally treated as fraction, p, of newborns vaccinated*

| | |
|---|---|
| at birth | HepB |
| 2 months | HepB (1-2 mos) + DTaP + PCV13 + Hib + Polio + RV |
| 4 months | DTaP + PCV13 + Hib + Polio + RV |
| 6 months | HepB (6-18 mos) + DTaP + PCV13 + Hib + Polio (6-18 mos) + RV |
| 12 Months | MMR (12-15 mos) + PCV13 (12-15 mos) + Hib (12-15 mos) + Varicella (12-15 mos) + HepA (12-23 mos) |
| 15 months | DTaP (15-18 mos) |



# 1. “Paediatric immunization”

- *Model this (as one time event)*

$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

$$\frac{dR}{dt} = \mu p + \gamma I - \mu R$$

- *Now what?*
- *Let’s derive expression for I**

# “Paediatric immunization”

- *After some algebra:*

$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

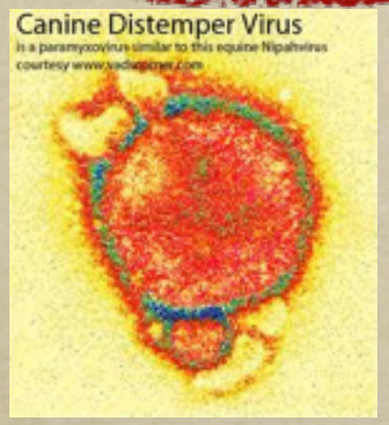
$$\frac{dR}{dt} = \mu p + \gamma I - \mu R$$

  - $I^* = \mu/\beta\ (R_0(1\text{-}p) - 1)$

- *Eradication implies $I^*=0$*
- *Requires $p = 1\text{-}1/R_0$*

  - *This is **fraction** of newborns to be immunized for (eventual) control*

# 2. Random Immunization




- *Consider wildlife diseases*
- *How would you vaccinate newborns?*
- *Pragmatically, will need continuous vaccination instead*

# "Random immunization"

- *After some algebra:*
  - $I^* = \mu/\beta\ (R_0\text{-}1 - \rho/\mu)$
- *Again, eradication* $\rightarrow I^*=0$
- *Requires* $\rho \geq \mu(R_0\text{-}1)$
- *This is* ***rate*** *of susceptibles to be immunized for (eventual) control*
- *What does criterion tell us, biologically?*

$$\frac{dS}{dt} = \mu - \beta SI - \mu S - \rho S$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

$$\frac{dR}{dt} = \rho S + \gamma I - \mu R$$

***Note****: at eradication threshold,* $\rho_c S^*$ *individuals vaccinated per unit time,*

$\mu(R_0\text{-}1) * 1/R_0$

$= \mu(1\text{-}1/R_0)$

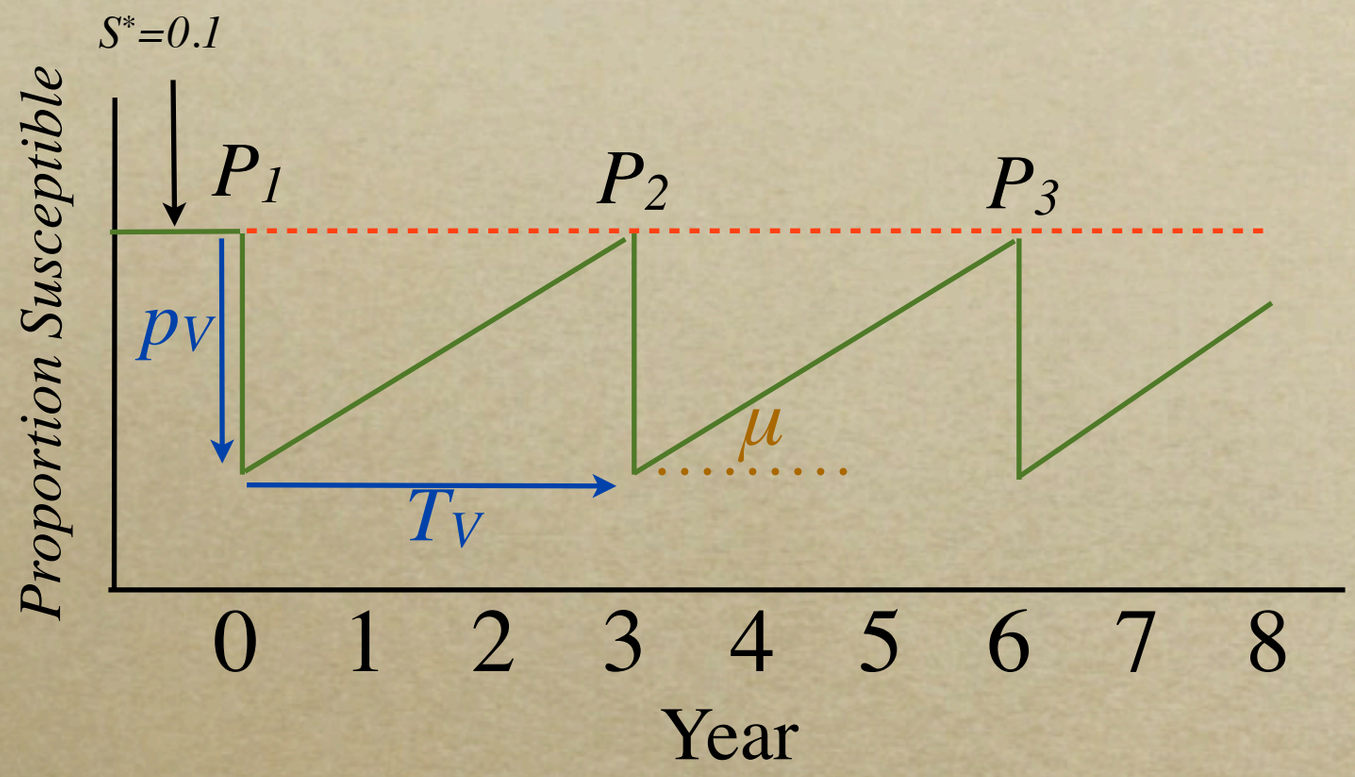
*Identical to infant immunization*

# 3. "Pulsed" Vaccination

- *Infant & Continuous vaccinations require sound infrastructure for vaccine delivery*
  - *unlikely to be case in many developing nations*

- *Alternative, perhaps more economic and logistically efficient strategy may be pulsed vaccination: immunize specific age cohorts at specified intervals*

# Pulsed Vaccination




- *Assume $R_0$ = 10*
- *$p_V$ = 60% and per capita annual birth rate = 2%*
- *For dI/dt<0, need to ensure S < 1/10*
- *After any pulse, S = 1/10 * 0.4 = 0.04*
- *Since μ=0.02, it'll take 3 years for S to reach 0.1*
- *So, pulse period = 3 yrs*

# More formally …

- *For an SIR model:*

*Vaccination fraction*

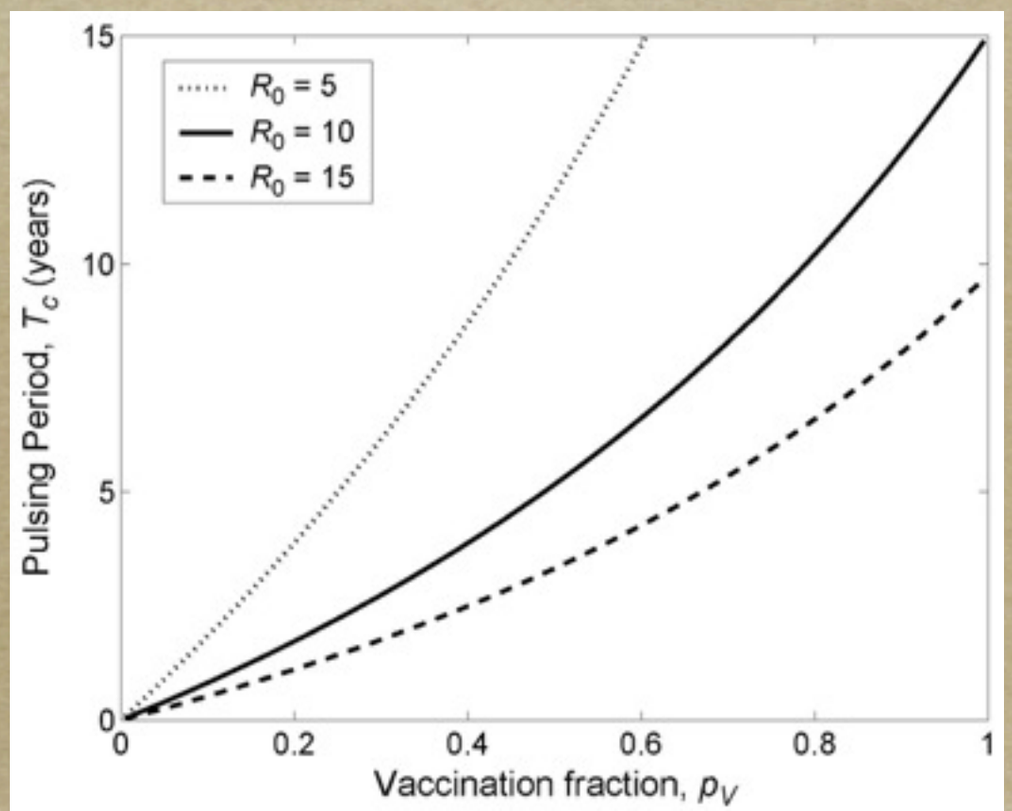
$$\frac{dS}{dt} = \mu - \beta SI - \mu S - p_V \sum_{n=0}^{\infty} S(nT^-)\delta(t - nT)$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

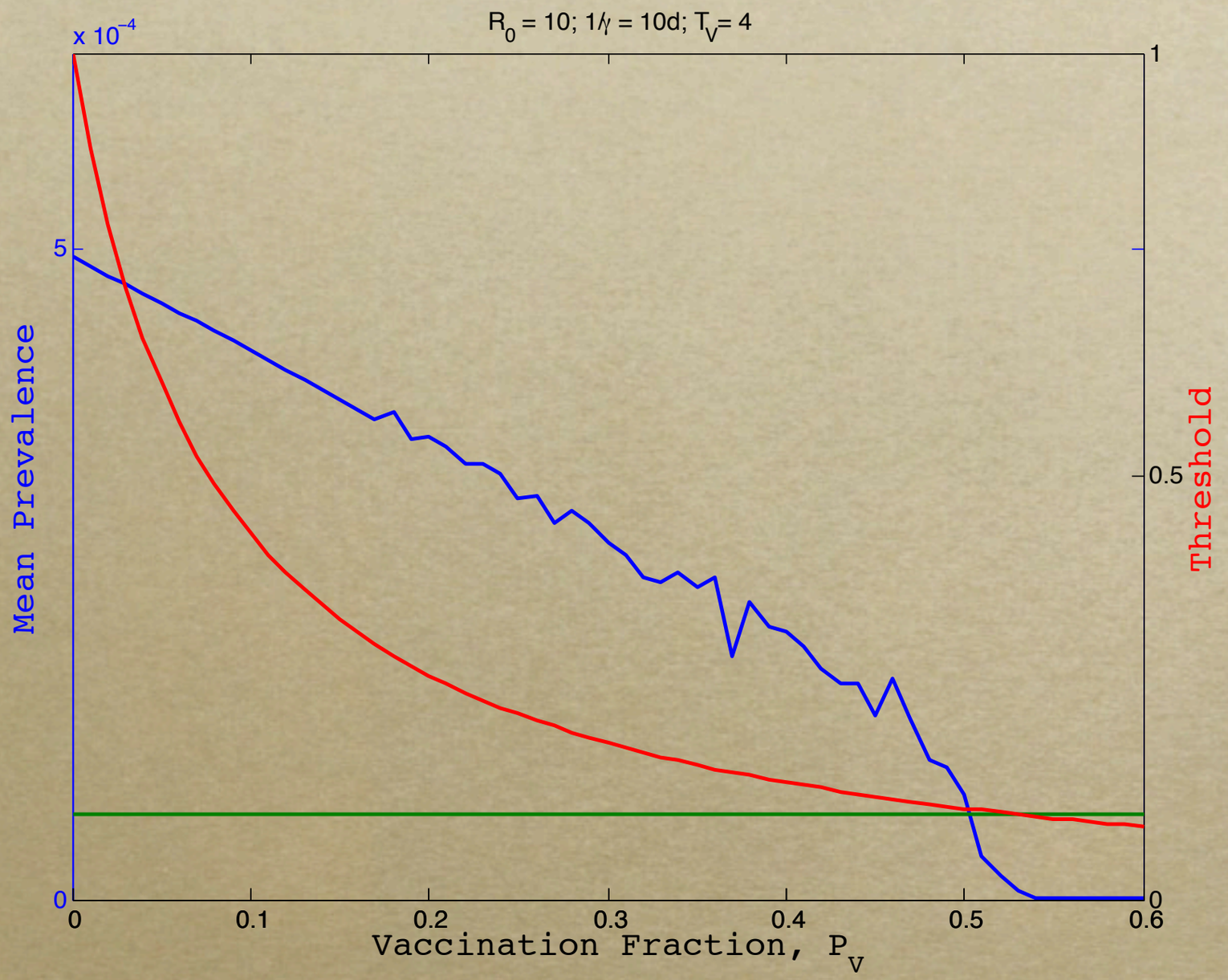
*Susceptibles prior to PV* *Dirac delta function*

- *Shulgin et al. (1998; Bull Math Biol): Linear stability analysis reveals eradication criterion*

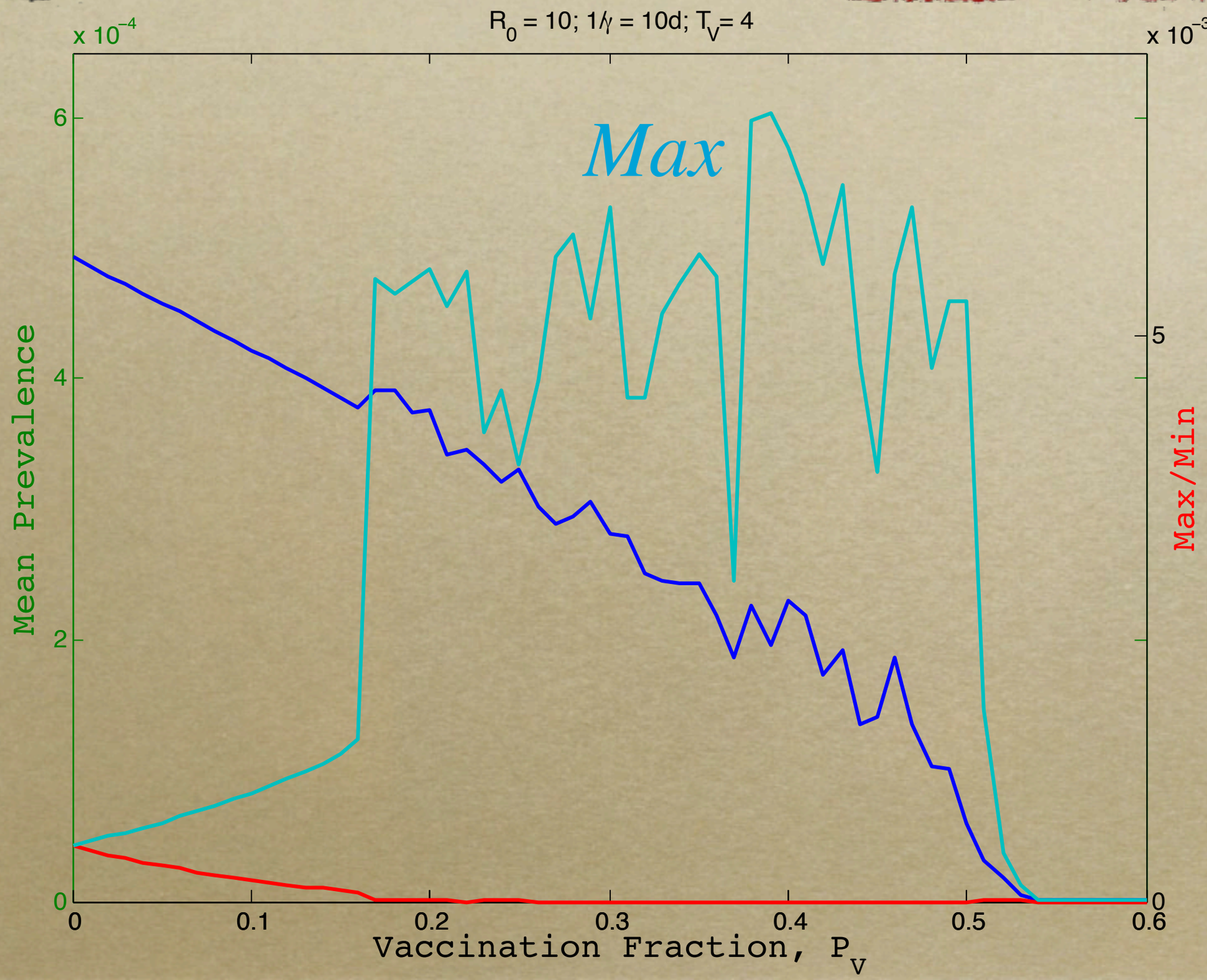
$$\frac{(\mu T - p_V)(e^{\mu T} - 1) + \mu p_V T}{\mu T(p_V - 1 + e^{\mu T})} < \frac{1}{R_0}$$



# Programming:

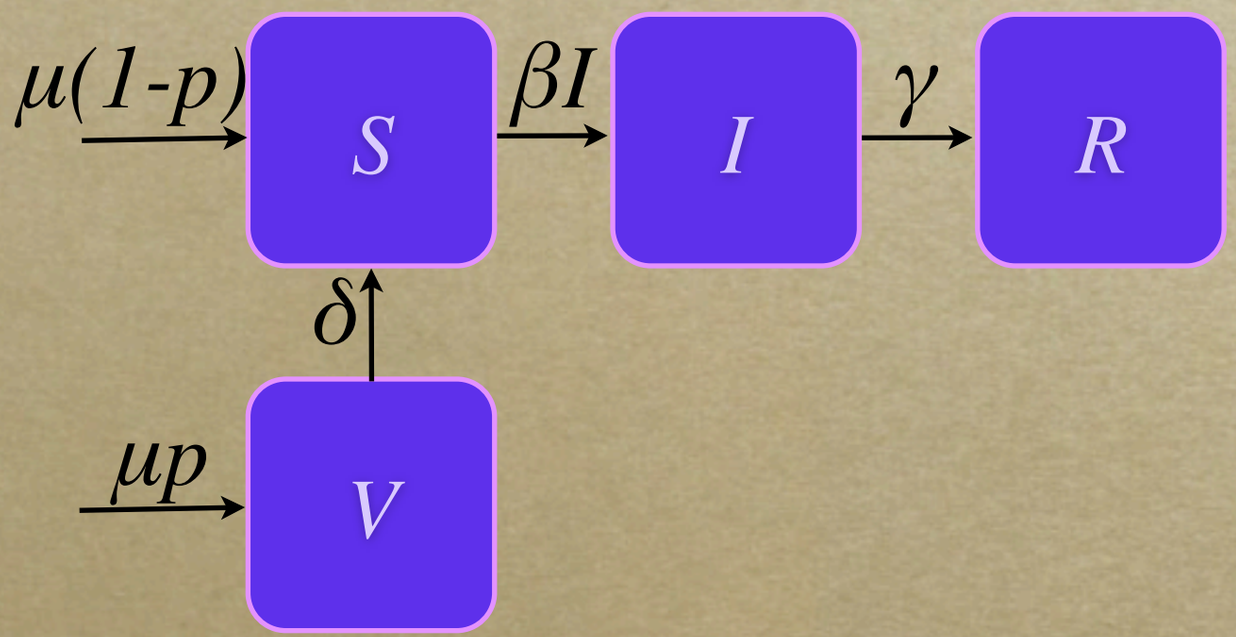


# Programming Challenge:



# Aside: Imperfect Vaccines

- *What if –as is at times the case– immunity derived from a vaccine wanes over time?*




$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S + \delta V$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

$$\frac{dV}{dt} = \mu p - (\mu + \delta)V$$

# Aside: Imperfect Vaccines

- *What if –as is at times the case– immunity derived from a vaccine wanes over time?*

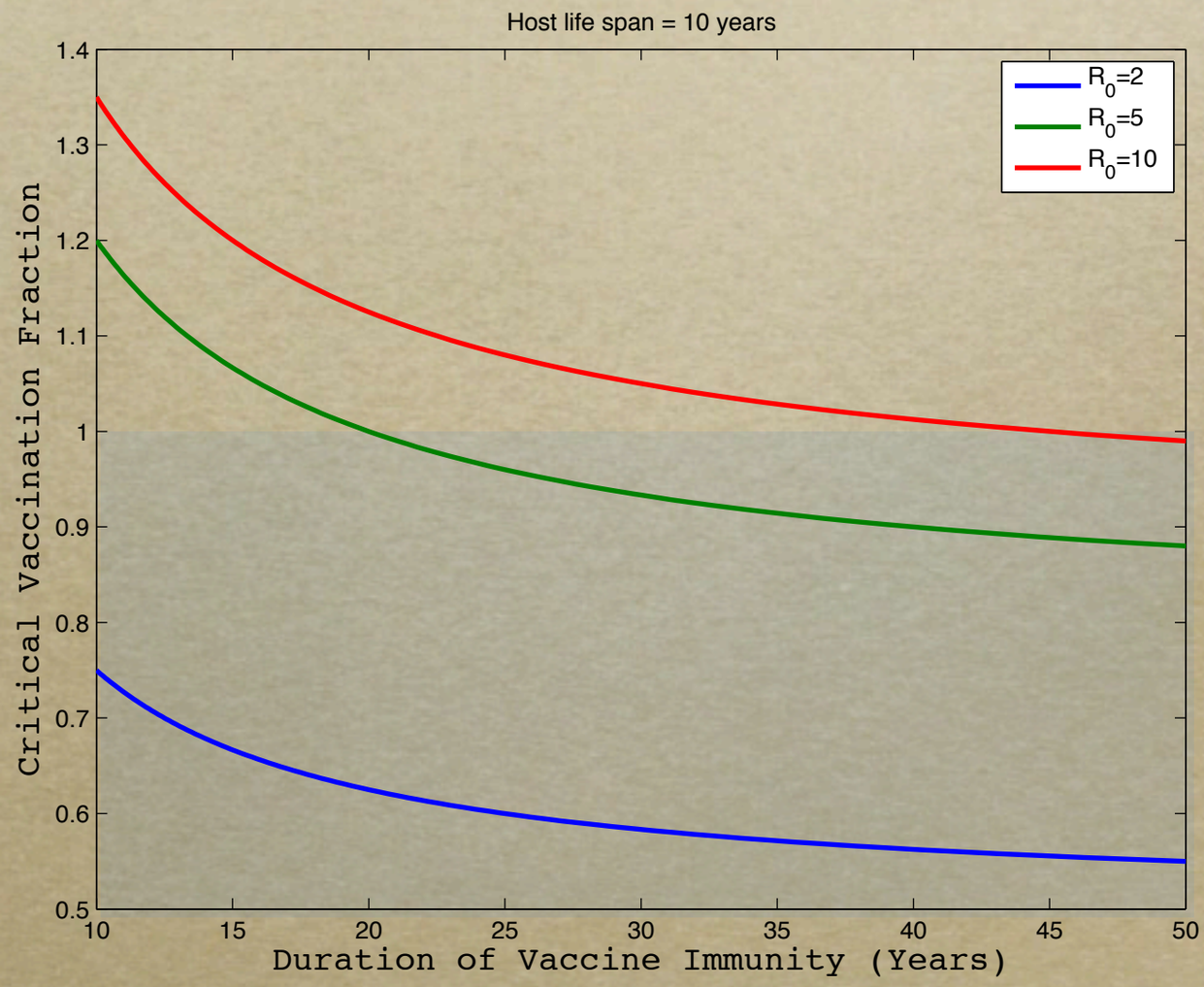
$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S + \delta V$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

$$\frac{dV}{dt} = \mu p - (\mu + \delta)V$$

*Eradication requires (Check this)*

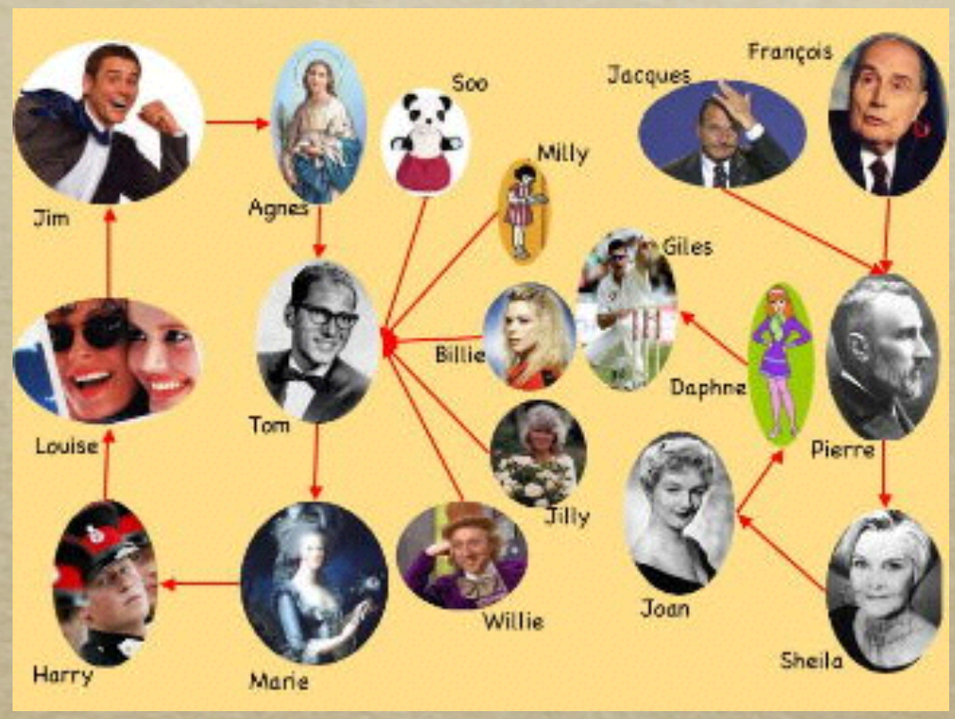
$$p = \left(1 - \frac{1}{R_0}\right)\left(1 + \frac{\delta}{\mu}\right)$$

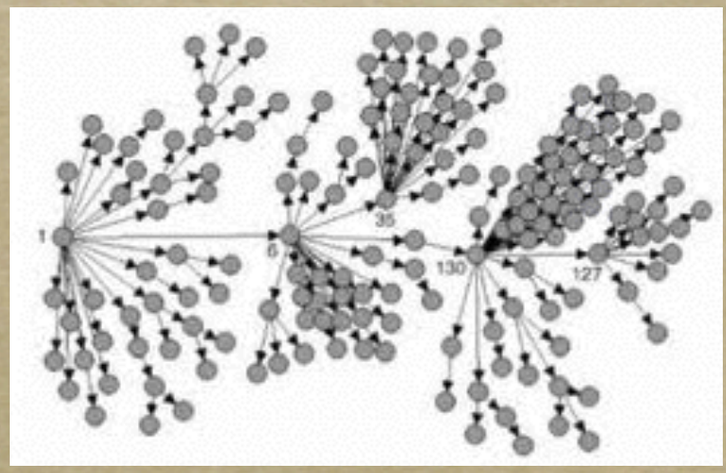



*Eradication will require boosters*

# 4. Non-Pharmaceutical Interventions

- *"Social distancing"*
- *Isolation and quarantining*
- *We should also find (or trace) their contacts*



# Background

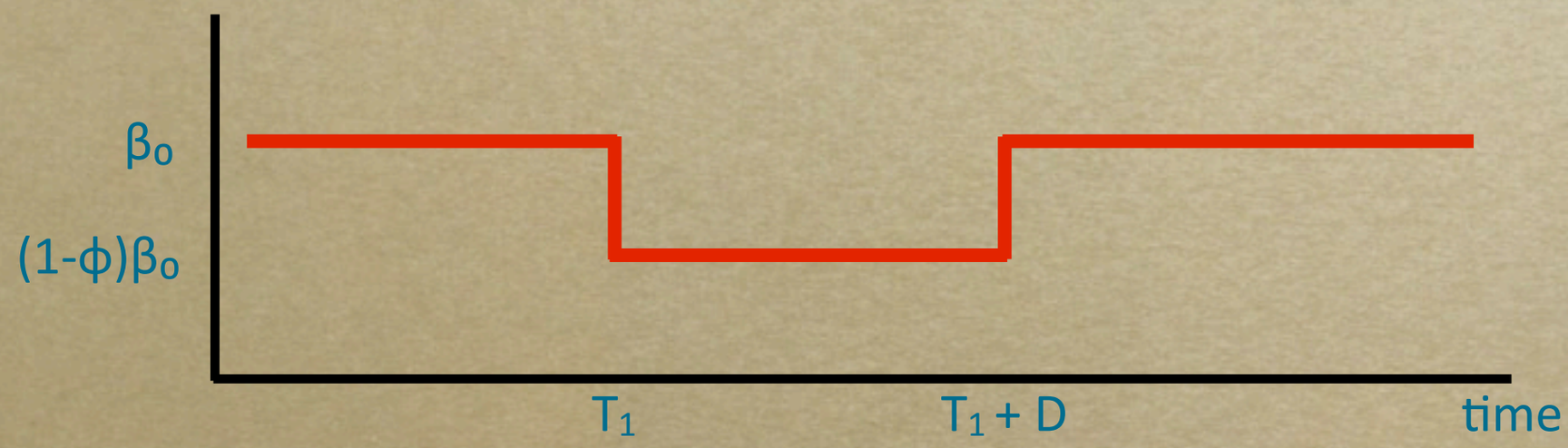
- Pandemic planning

  Consider emerging pathogen
  Everyone susceptible
  No pharmaceutical defense (drugs/vaccines)
  Only Non-Pharmaceutical Interventions would work

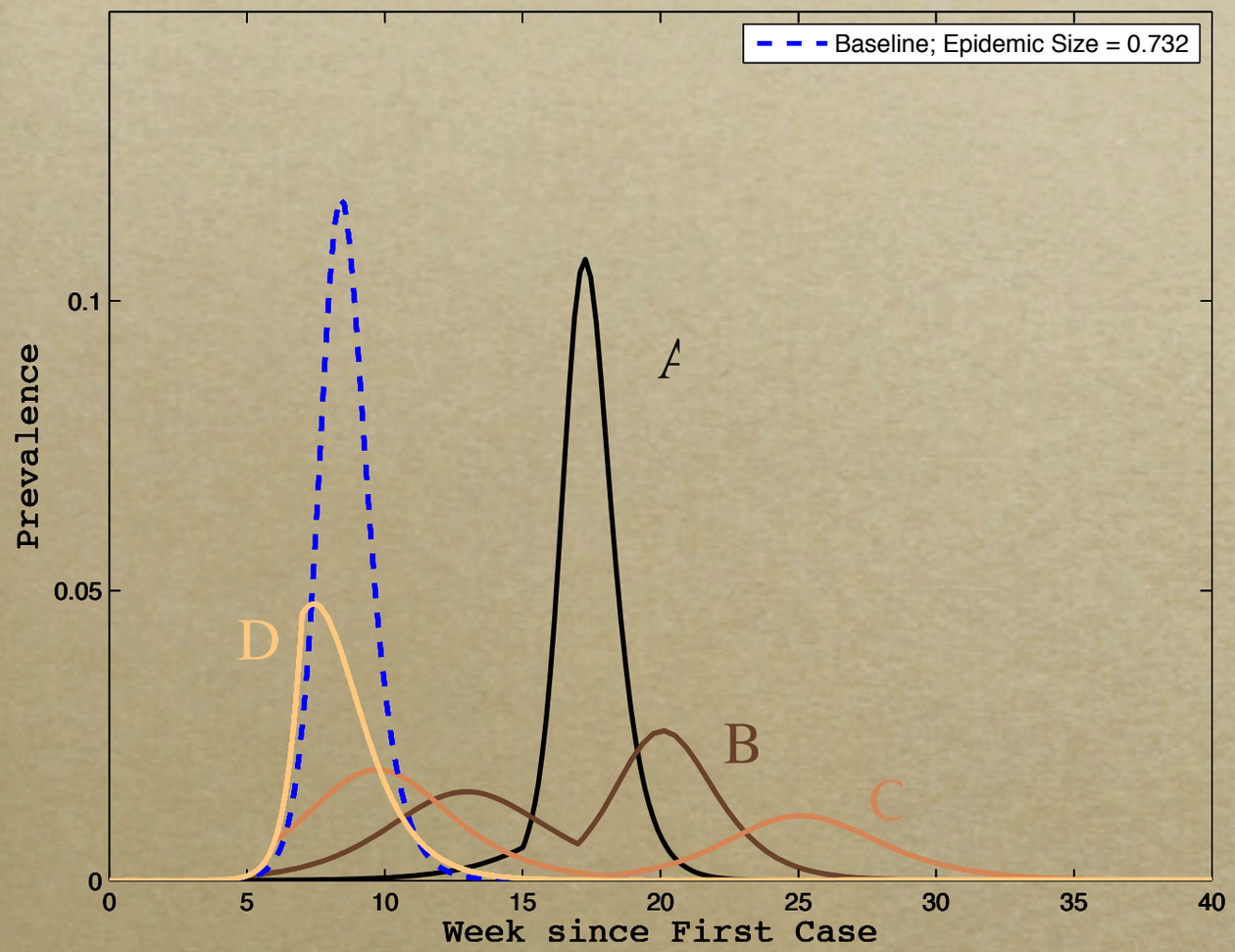
Social distancing
  How long?
  What extent?

# Protocol

- Basic reproduction ratio $R_0 = 1.8$
- Recovery rate $\gamma = 1/2.6$ day$^{-1}$
  - Generation time 2.6 days
- Baseline transmission rate $\beta_0 = R_0 \gamma$
- Population size $n = 58.1$ million (UK)



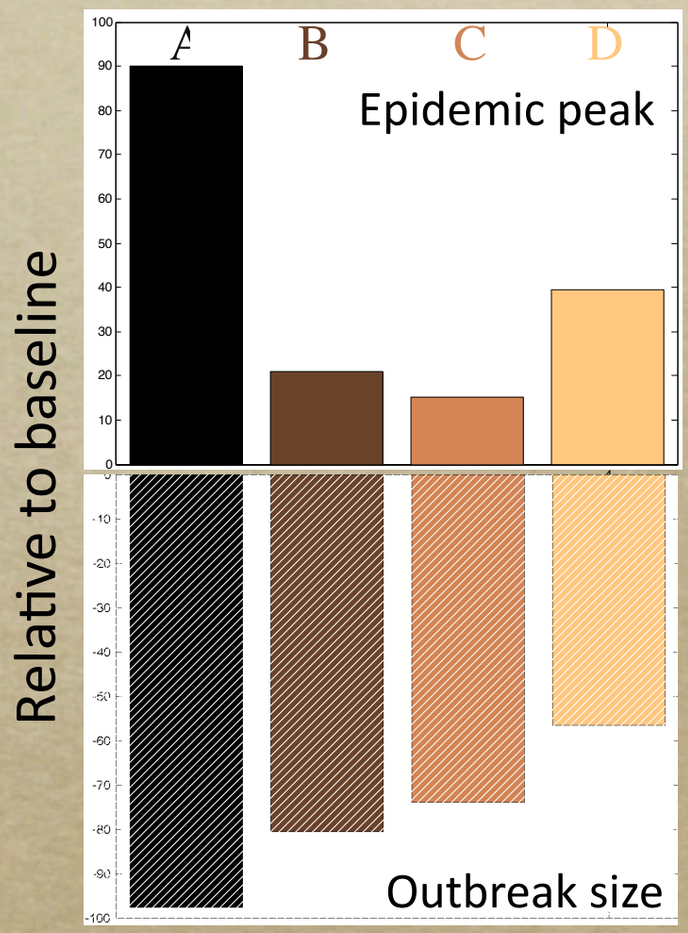
# Intervention D=12 weeks



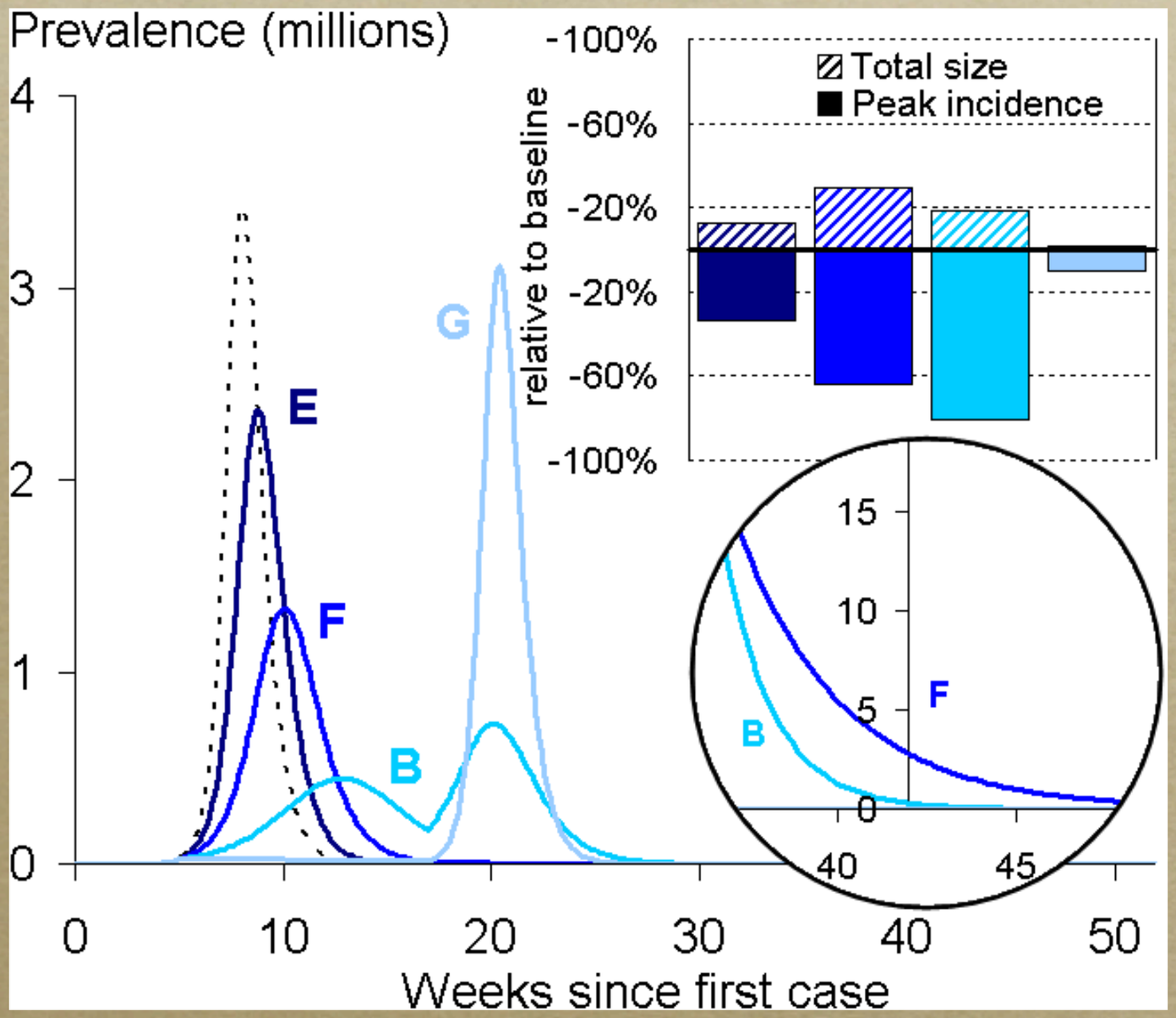

Intervention $\phi = 0.333$

Start (week)

- A: $T_1 = 3$
- B: $T_1 = 5$
- C: $T_1 = 6$
- D: $T_1 = 7$



# Intervention D=12 weeks




- Start (week)
  $T_1 = 5$
- Intervention

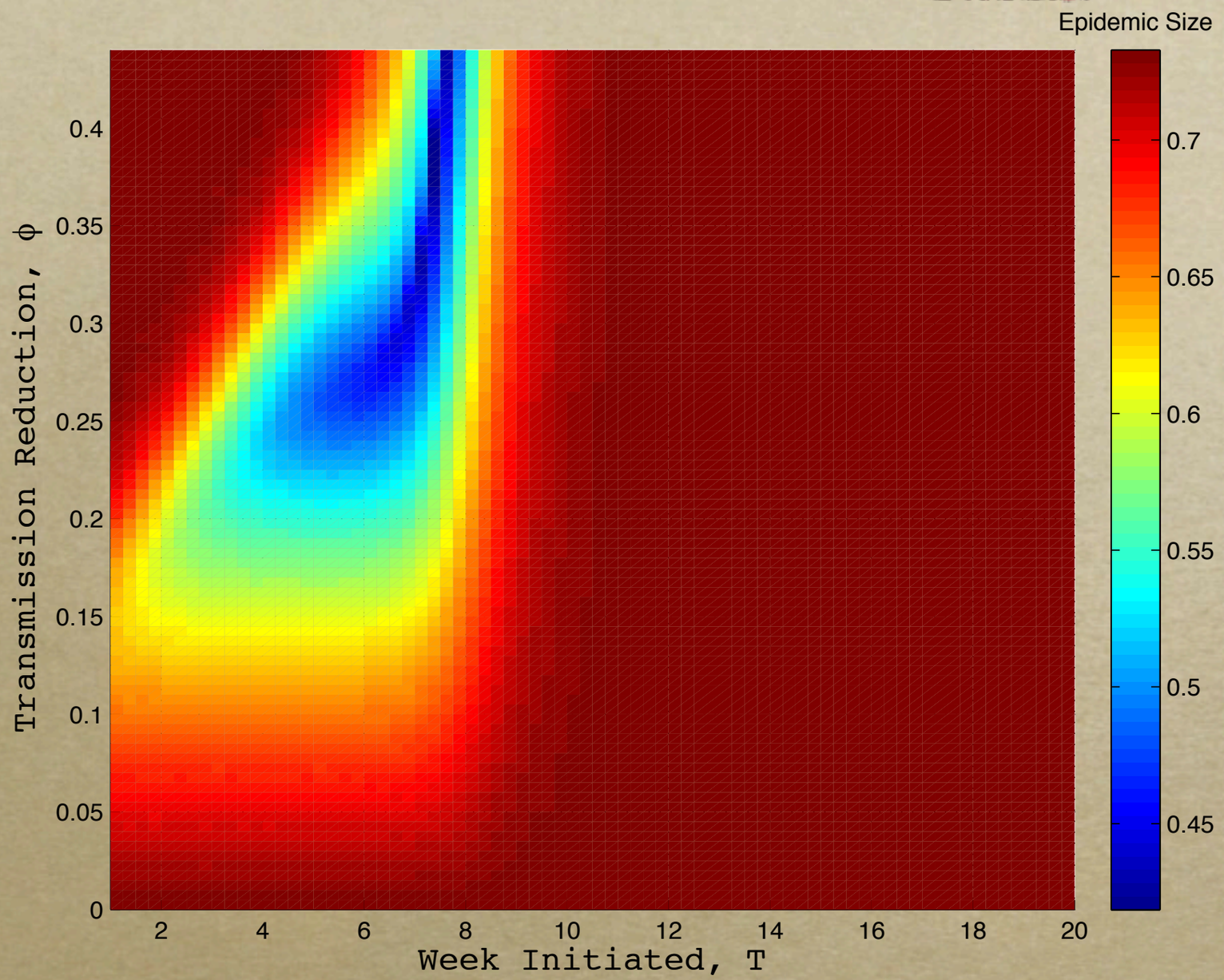
E: $\phi = 0.111$

F: $\phi = 0.222$

B: $\phi = 0.333$

G: $\phi = 0.444$

# Intervention D=12 weeks




*Depending on aims of control, efforts that are too early or too severe may be counter-productive*

# Contact-tracing & isolation

- *Assume average contact rate*, $\kappa$
- *Transmission probability*, $\nu$
- *Infectious individuals immediately symptomatic*
- *Infectious isolated at rate* $d_I$
- *Fraction q of contacts with infectious quarantined*
- *Kept in quarantine for average* $\tau_Q$

# Modeling NPI

*Contact, transmission*

*Contact, no transmission, quarantine*

*Released quarantine*

$$\frac{dS}{dt} = -\left(\kappa\nu + q\kappa(1-\nu)\right)SI + \tau_Q S_Q$$

*Susceptible*

*Susceptible in quarantine*

*Infectious*

*Infectious in quarantine*

*Recovered*

*Contact, transmission, not traced*

*Isolation*

*Contact, transmission, traced*

# Modeling NPI

*Contact, no transmission, quarantine*

*Contact, transmission*

*Released quarantine*

*Susceptible*

$$\frac{dS}{dt} = -\left(\kappa\nu + q\kappa(1-\nu)\right)SI + \tau_Q S_Q$$

*Susceptible in quarantine*

$$\frac{dS_Q}{dt} = q\kappa(1-\nu)SI - \tau_Q S_Q$$

*Contact, transmission, not traced*

*Infectious*

$$\frac{dI}{dt} = \kappa\nu(1-q)SI - d_I I - \gamma I$$

*Isolation*

*Infectious in quarantine*

$$\frac{dQ}{dt} = \kappa\nu q SI + d_I I - \tau_Q Q$$

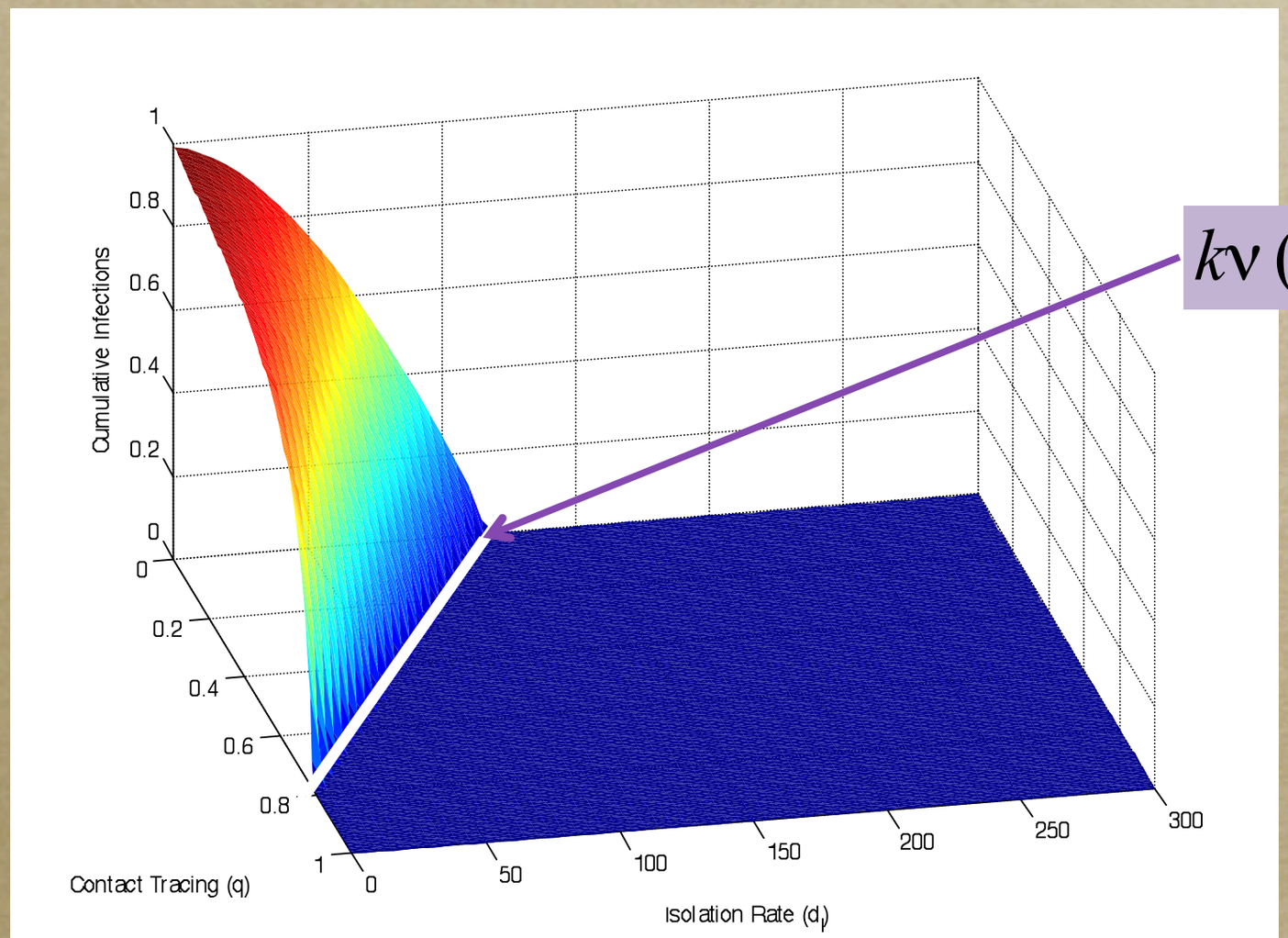
*Contact, transmission, traced*

*Recovered*

$$\frac{dR}{dt} = \gamma I + \tau_Q Q$$

# What does it tell us?

- *Can show control requires* $S < \dfrac{(d_I + \gamma)}{k\nu(1-q)}$




$R_0 = 5$

$\tau_Q = 21\ d$

# Yes, but …

***Key realities we've ignored:***

1. *Assumed infectious individuals immediately symptomatic (often, clinical presentation a few days after infectiousness, eg SARS)*
2. *Uncertainties & delays in identifying and isolating potential contacts*

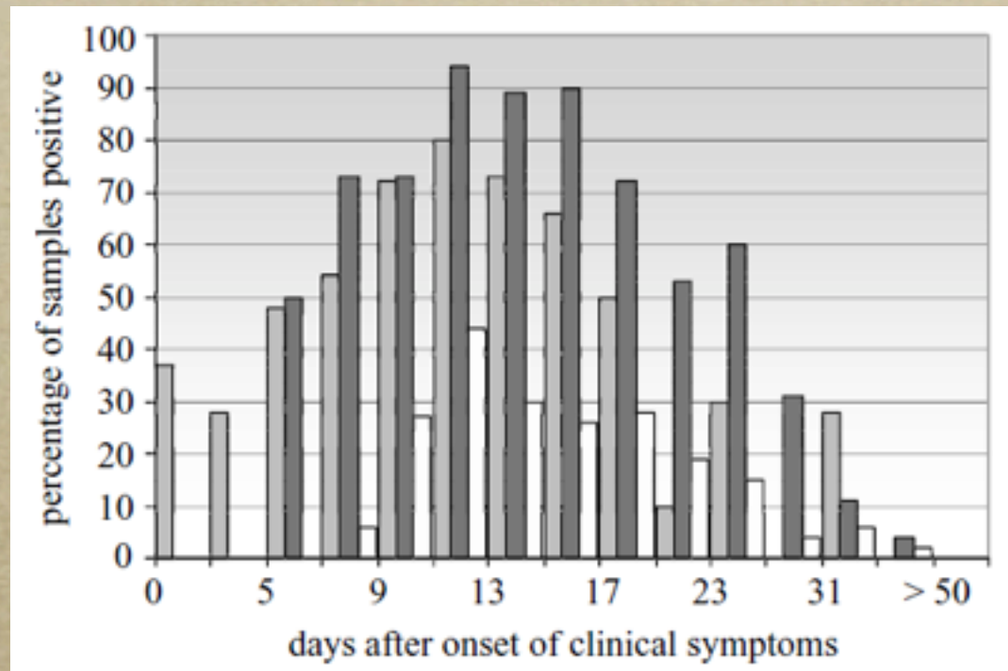
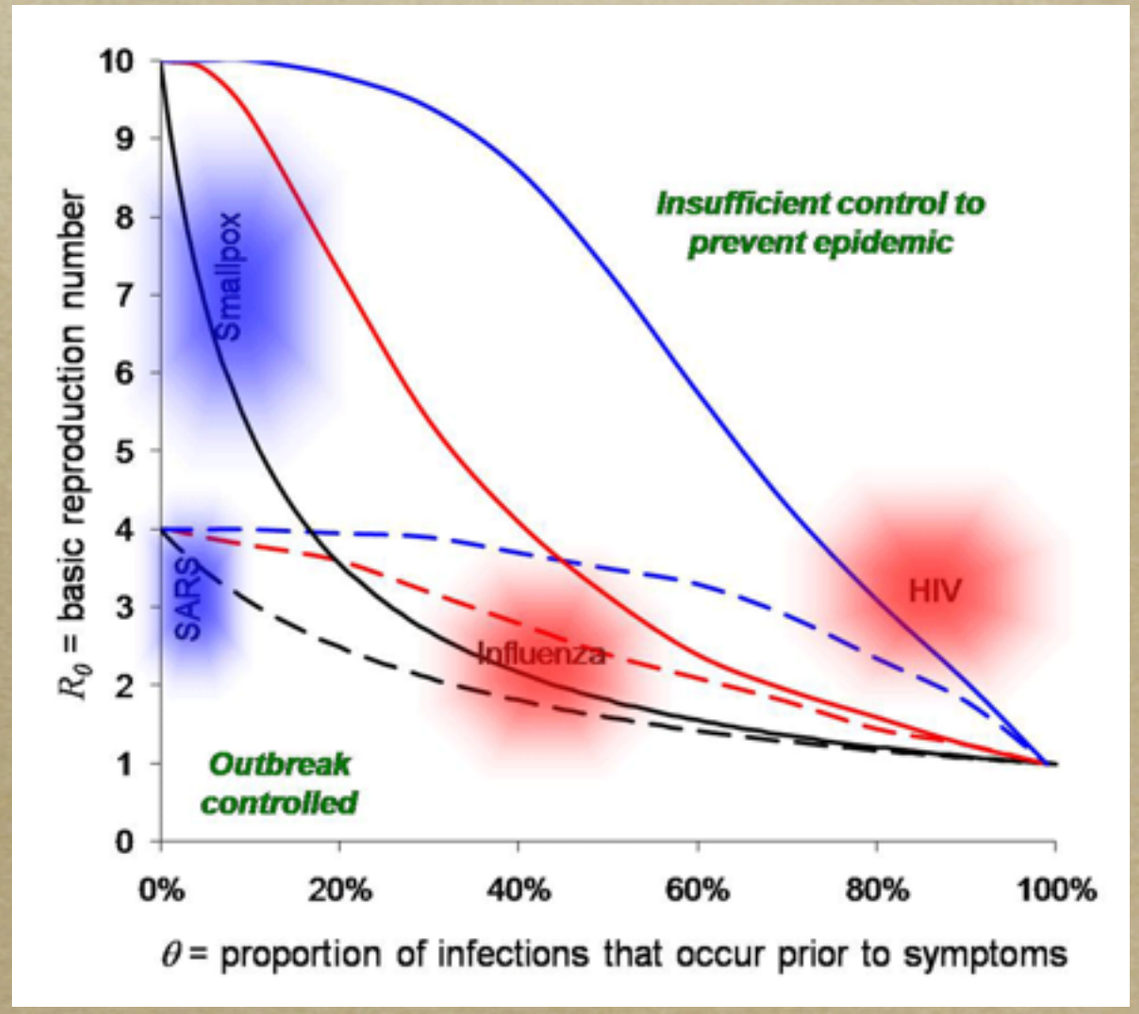



Figure 4. Studies of viral shedding in SARS patients on various days following the onset of clinical symptoms, in stool (dark-grey bars), urine (white bars) and naso-pharangeal aspirate (light-grey bars) (Peiris *et al.* 2003*a*).

# Yes, but …

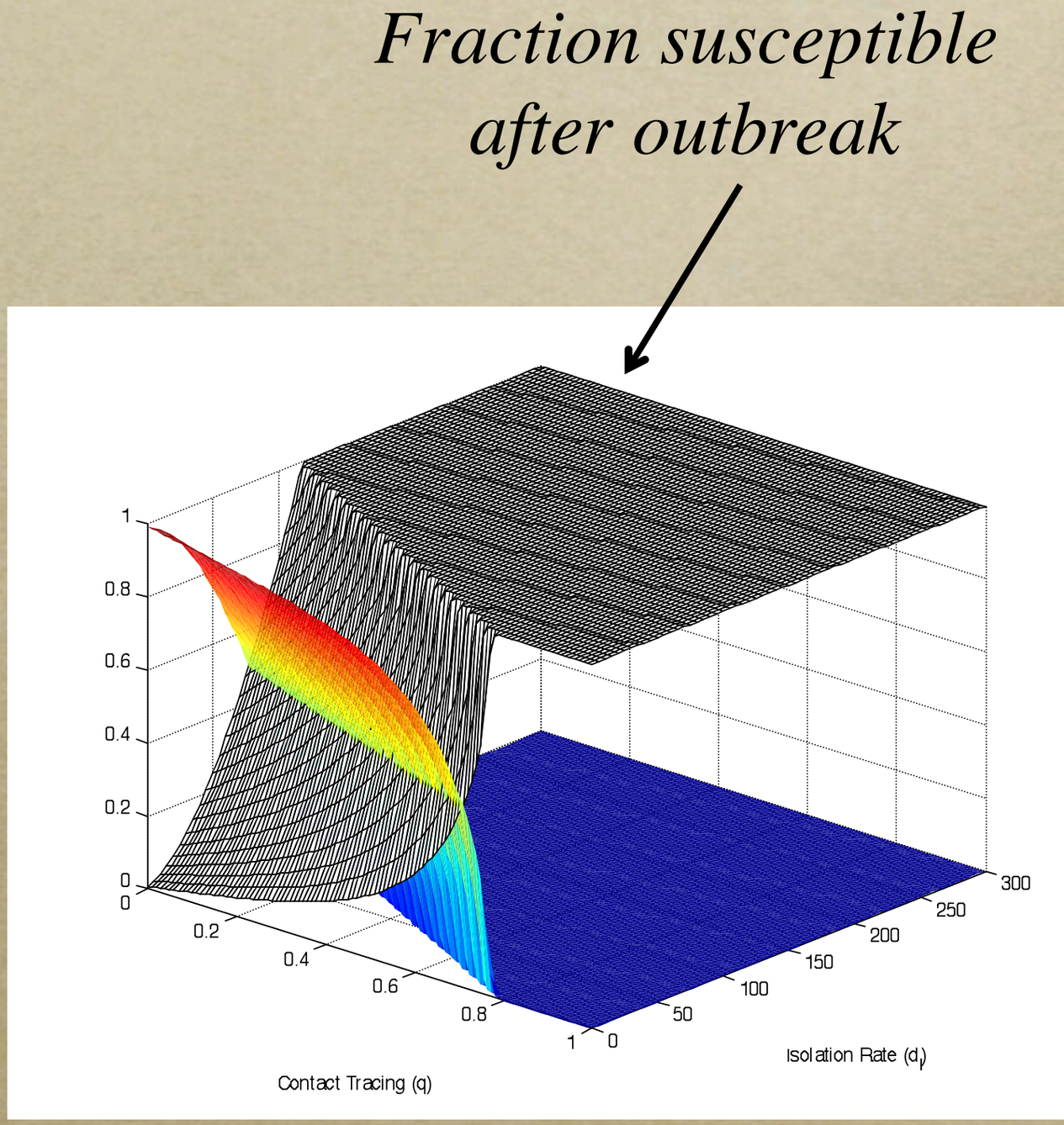
- *Fraser et al. (2004; PNAS) examined 'controllability' of an infectious disease, based on epidemiology and pathogenesis*

*Infectious disease with much 'silent' transmission are harder to control this way*



# Yes, another but …

- ***Back to our NPI example:***
  - *If contact tracing and quarantining efficient enough, invasion can be controlled*
  - *But …*
  - *Let's consider remaining susceptible population, post-control*
  - *NPI measures leave population vulnerable to re-exposure*



# Lecture Summary …

- *Models can generate predictions about immunization levels required for eradication*
- *Similarly, extent of non-pharmaceutical interventions can be gauged*
- *NPIs leave many susceptibles behind*
  - *Important for re-introductions*
- *Infections with much silent transmission very difficult to control with NPIs*