# Inference in Randomized Trials with Death and Missingness SISCR Shortcourse 

Daniel Scharfstein<br>Johns Hopkins University dscharf@jhu.edu

$$
\text { July 26, } 2016
$$

## HT-ANAM 302 Study

- Anamorelin is a drug developed for the treatment of cancer cachexia and anorexia.
- HT-ANAM 302 was a randomized, double-blind, placebo-controlled Phase III study designed to evaluate the efficacy of anamorelin in patients with advanced non-small cell lung cancer.
- Lean body mass (LBM) was scheduled to be measured at baseline $\left(Y_{0}\right), 6$ weeks $\left(Y_{1}\right)$ and 12 weeks $\left(Y_{2}\right)$
- Primary functional endpoint: $Z=\frac{\left(Y_{2}+Y_{1}\right)}{2}-Y_{0}$


## Death and missingness

|  | Placebo | Anamorelin |
| ---: | :---: | :---: |
|  | $n=157$ | $n=322$ |
| Died Prior to Wk 12 | $24(15.3 \%)$ | $54(16.8 \%)$ |
| Survivors with complete data | $93(59.2 \%)$ | $185(57.5 \%)$ |
| Survivors missing only Wk 6 | $3(1.9 \%)$ | $17(5.3 \%)$ |
| Survivors missing only Wk 12 | $17(10.8 \%)$ | $31(9.6 \%)$ |
| Survivors missing both Wks 6, 12 | $20(12.7 \%)$ | $35(10.9 \%)$ |

## Central Question

How should data from studies like HT-ANAM 302 be analyzed to evaluate the effect of treatment on the functional outcome?

## Key Issue

- Distinction between missing data and data truncated by death
- Missing data: exist but not collected
- Data truncated by death: does not exist and undefined
- Can't just treat as a missing data problem.


## Common Approaches

(1) Evaluate treatment effect on functional outcome conditional on survival

- Conditioning on post-baseline factor
(2) Joint modeling survival and functional outcomes
- Allows extrapolation of outcomes after death
(3) Principal stratification
- Applies to a subset of patients who are not identifiable at baseline
(4) Composite endpoint combining survival and functional outcomes
- May be hard to separate effect on function.


## Bottom Line

## NO PERFECT SOLUTIONS

Not a fan of Approaches 1 and 2.

## Goal

To construct a composite endpoint approach that handles both death and missing data

## Notation

- $T=0,1$ : treatment assignment
- $X$ vector baseline covariates
- $Y_{0}$ : baseline functional measure at $t_{0}$
- $Y_{1}, \ldots, Y_{K}$ : functional outcomes at $t_{1}, \ldots, t_{K}$
- $L$ : survival time
- $A_{k}=I\left(L>t_{k}\right)$ : survival status at $t_{k}$
- $Z=g\left(Y_{0}, \ldots, Y_{K}\right)$ : primary functional endpoint
- e.g. $K=2, Z=\left(Y_{2}+Y_{1}\right) / 2-Y_{0}$
- only defined when $A_{K}=1$


## Composite Outcome

Finite-valued random variable $U$ which assigns a score to each patient such that

- each patient who dies prior to $t_{K}$ is assigned a score according to their survival time $(L)$, with shorter survival times assigned lower scores
- each patient who survives past $t_{K}$ is assigned a score (higher than those who died prior to $t_{K}$ ) according to their functional status $(Z)$, with lower functional status assigned lower scores.
Only the ordering of $U$ is important, not the actual score assignments.


## Mathematical Definition

- Let $W=L$ if $A_{K}=0$ and $W=Z$ if $A_{K}=1$
- $U$ is a function of $\left(A_{K}, W\right)$
- $U$ is defined such that
- For all $\omega \in \Omega, U(\omega)<c$ when $A_{K}(\omega)=0$
- For all $\omega, \omega^{\prime} \in \Omega$

$$
\begin{array}{ll}
U(\omega)<U\left(\omega^{\prime}\right) & \text { if } A_{K}(\omega)=A_{K}\left(\omega^{\prime}\right), W(\omega)<W\left(\omega^{\prime}\right) \\
U(\omega)>U\left(\omega^{\prime}\right) & \text { if } A_{K}(\omega)=A_{K}\left(\omega^{\prime}\right), W(\omega)>W\left(\omega^{\prime}\right) \\
U(\omega)=U\left(\omega^{\prime}\right) & \text { if } A_{K}(\omega)=A_{K}\left(\omega^{\prime}\right), W(\omega)=W\left(\omega^{\prime}\right) \\
U(\omega)<U\left(\omega^{\prime}\right) & \text { if } A_{K}(\omega)=0, A_{K}\left(\omega^{\prime}\right)=1 \\
U(\omega)>U\left(\omega^{\prime}\right) & \text { if } A_{K}(\omega)=1, A_{K}\left(\omega^{\prime}\right)=0 .
\end{array}
$$

## Ranking examples

- $A_{K, i}=A_{K, j}=1$
- $Z_{i}>Z_{j}$ : subject $i$ ranked better than subject $j$
- $Z_{i}<Z_{j}$ : subject $j$ ranked better than subject $i$
- $Z_{i}=Z_{j}$ : subjects $i$ and $j$ ranked the same
- $A_{K, i}=A_{K, j}=0$
- $L_{i}>L_{j}$ : subject $i$ ranked better than subject $j$
- $L_{i}<L_{j}$ : subject $j$ ranked better than subject $i$
- $L_{i}=L_{j}$ : subjects $i$ and $j$ ranked the same
- $A_{K, i}=1, A_{K, j}=0$
- subject $i$ ranked better than subject $j$
- $A_{K, i}=0, A_{K, j}=1$
- subject $j$ ranked better than subject $i$


## Treatment Effect

Treatment effect $(\theta)$ is measured by the probability that the outcome for an individual with $T=0$ is less than the outcome of an individual with $T=1$ minus the probability that the outcome for an individual with $T=0$ is greater than the outcome of an individual with $T=1$

- $\theta=0$ under the null
- $\theta>0$ favors $T=1 ; \theta<0$ favors $T=0$
- First part: Mann-Whitney
- Second part: needed to handle ties

Can also compare the treatment-specific quantiles of $U$.

## Estimation of $\theta$

In the absence of missing data,

$$
\widehat{\theta}=\frac{1}{n_{0} n_{1}} \sum_{i: T_{i}=0} \sum_{j: T_{j}=1}\left\{I\left(U_{i}<U_{j}\right)-I\left(U_{i}>U_{j}\right)\right\}
$$

where $n_{0}=\sum_{i}\left(1-T_{i}\right)$ and $n_{1}=\sum_{i} T_{i}$.

## Missing Data

- $R_{k}$ : missing data indicator (defined when $A_{k}=1$ )
- $S=\left(R_{1}, \ldots, R_{K}\right)$ (defined when $A_{K}=1$ )
- $Y_{o b s}^{(s)}=\left\{Y_{k}: R_{k}=1, k \geq 1, S=s\right\}$
- $Y_{\text {mis }}^{(s)}=\left\{Y_{k}: R_{k}=0, k \geq 1, S=s\right\}$
- $Z$ is unobserved when $S \neq 1$.

To estimate $\theta$, need to impute $Z$ or equivalently $Y_{\text {mis }}^{(s)}$ for $s \neq \mathbf{1}$

## Observed Data



## Missing Data Assumptions

$$
\begin{aligned}
& f\left(Y_{m i s}^{(s)} \mid A_{K}=1, Y_{o b s}^{(s)}, Y_{0}, X, T, S=s\right) \\
& \propto \exp \left(\beta_{T} Z\right) \underbrace{f\left(Y_{m i s}^{(s)} \mid A_{K}=1, Y_{o b s}^{(s)}, Y_{0}, X, T, S=\mathbf{1}\right)}_{\text {Reference Distribution }}
\end{aligned}
$$

for all $s \neq \mathbf{1}$,

- $\beta_{T}$ is a treatment-specific sensitivity parameter.
- $\beta_{T}=0$ (i.e., benchmark assumption) reduces to the complete case missing value (CCMV) restrictions applied to the missing data patterns for patients alive at $t_{K}$.
- CCMV is different than missing at random (MAR) assumption.


## HT-ANAM 302 Study

- $K=2, Z=\left(Y_{1}+Y_{2}\right) / 2-Y_{0}$.
- $\beta_{T}^{\prime}=2 \beta_{T}$

$$
\begin{aligned}
& f\left(Y_{2} \mid A_{2}=1, Y_{1}, Y_{0}, X, T, S=(1,0)\right) \\
& \propto \exp \left(\beta_{T}^{\prime} Y_{2}\right) \underbrace{f\left(Y_{2} \mid A_{2}=1, Y_{1}, Y_{0}, X, T, S=\mathbf{1}\right)}_{\text {Reference Distribution }}
\end{aligned}
$$

For subjects alive at $t_{2}$, who are observed at time $t_{1}$, who share the same functional measure at $t_{1}$ and who share the same baseline factors, the distribution of $Y_{2}$ for those whose functional measure at $t_{2}$ is missing is, when $\beta_{T}^{\prime}>0(<0)$, more heavily weighted toward higher (lower) values of $Y_{2}$ than those whose functional measure at $t_{2}$ is observed.

## HT-ANAM 302 Study

$$
\begin{aligned}
& f\left(Y_{1} \mid A_{2}=1, Y_{2}, Y_{0}, X, T, S=(0,1)\right) \\
& \propto \exp \left(\beta_{T}^{\prime} Y_{1}\right) \underbrace{f\left(Y_{1} \mid A_{2}=1, Y_{2}, Y_{0}, X, T, S=1\right)}_{\text {Reference Distribution }}
\end{aligned}
$$

For subjects alive at $t_{2}$, who are observed at time $t_{2}$, who share the same functional measure at $t_{2}$ and who share the same baseline factors, the distribution of $Y_{1}$ for those whose functional measure at $t_{1}$ is missing is, when $\beta_{T}^{\prime}>0(<0)$, more heavily weighted toward higher (lower) values of $Y_{1}$ than those whose functional measure at $t_{1}$ is observed.

## HT-ANAM 302 Study

$$
\begin{aligned}
& f\left(Y_{1}, Y_{2} \mid A_{2}=1, Y_{0}, X, T, S=(0,0)\right) \\
& \quad \propto \exp \left(\beta_{T}^{\prime}\left(Y_{1}+Y_{2}\right)\right) \underbrace{f\left(Y_{1}, Y_{2} \mid A_{2}=1, Y_{0}, X, T, S=1\right)}_{\text {Reference Distribution }}
\end{aligned}
$$

For subjects alive at $t_{2}$ and who share the same baseline factors, the joint distribution of $Y_{1}$ and $Y_{2}$ for those whose functional measures at $t_{1}$ and $t_{2}$ are missing is, when $\beta_{T}^{\prime}>0$ $(<0)$, more heavily weighted toward higher (lower) values of $Y_{1}$ and $Y_{2}$ than those whose measures are fully observed.

## HT-ANAM 302 Study

- Ignore conditioning on $Y_{0}$ and $X$ and suppose $f\left(Y_{1}, Y_{2} \mid A_{2}=1, T, S=\mathbf{1}\right)$ is multivariate normal with mean $\left(\mu_{T, 1}, \mu_{T, 2}\right)$ and variance-covariance matrix

$$
\Sigma_{T}=\left[\begin{array}{ll}
\sigma_{T, 1}^{2} & \rho_{T} \sigma_{T, 1} \sigma_{T, 2} \\
\rho_{T} \sigma_{T, 1} \sigma_{T, 2} & \sigma_{T, 2}^{2}
\end{array}\right]
$$

- $f\left(Y_{2} \mid A_{2}=1, Y_{1}, T, S=(1,0)\right)$ is normal with mean $\mu_{T, 2}+\beta_{T}^{\prime}\left(1-\rho_{T}^{2}\right) \sigma_{T, 2}^{2}+\rho_{T} \frac{\sigma_{T, 2}}{\sigma_{T, 1}}\left(Y_{1}-\mu_{T, 1}\right)$ and variance $\left(1-\rho_{T}^{2}\right) \sigma_{T, 2}^{2}$
- $f\left(Y_{1} \mid A_{2}=1, Y_{2}, T, S=(0,1)\right)$ is normal with mean $\mu_{T, 1}+\beta_{T}^{\prime}\left(1-\rho_{T}^{2}\right) \sigma_{T, 1}^{2}+\rho_{T} \frac{\sigma_{T, 1}}{\sigma_{T, 2}}\left(Y_{2}-\mu_{T, 2}\right)$ and variance $\left(1-\rho_{T}^{2}\right) \sigma_{T, 1}^{2}$


## HT-ANAM 302 Study

- $f\left(Y_{1}, Y_{2} \mid A_{2}=1, T, S=(0,0)\right)$ is multivariate normal with mean $\left(\mu_{T, 1}+\beta_{T}^{\prime} \sigma_{T, 1}^{2}+\beta_{T}^{\prime} \rho_{T} \sigma_{T, 1} \sigma_{T, 2}, \mu_{T, 2}+\right.$ $\left.\beta_{T}^{\prime} \sigma_{T, 2}^{2}+\beta_{T}^{\prime} \rho_{T} \sigma_{T, 1} \sigma_{T, 2}\right)$ and variance-covariance matrix $\Sigma_{T}$.
- If $\rho_{T}>0$, then the means increase linearly in $\beta_{T}^{\prime}$
- $\beta_{T}^{\prime}$ has no impact on the variances and covariances.
- $\beta_{T}^{\prime}>0\left(\beta_{T}^{\prime}<0\right)$ implies that the non-identified distributions have more (less) mass at higher values than their reference distributions.


## Example: Exponential tilting



## Modeling

Need to specify of a model for

$$
f\left(\bar{Y}_{K} \mid A_{K}=1, Y_{0}, X, T, S=\mathbf{1}\right)
$$

- To respect bounds, define

$$
\phi\left(y_{k}\right)=\log \left\{\frac{y_{k}-B_{L}}{B_{U}-y_{k}}\right\}
$$

- $Y_{k}^{\dagger}=\phi\left(Y_{k}\right)$ and $\bar{Y}_{k}^{\dagger}=\left(Y_{1}^{\dagger}, \ldots, Y_{k}^{\dagger}\right)$.
- One-to-one mapping between

$$
h\left(\bar{Y}_{K}^{\dagger} \mid A_{K}=1, Y_{0}, X, T, S=\mathbf{1}\right)
$$

and

$$
f\left(\bar{Y}_{K} \mid A_{K}=1, Y_{0}, X, T, S=\mathbf{1}\right)
$$

## Modeling

$$
\begin{aligned}
& h\left(\bar{Y}_{K}^{\dagger} \mid A_{K}=1, Y_{0}, X, T, S=\mathbf{1}\right)= \\
& \quad \prod_{k=1}^{K} h\left(Y_{k}^{\dagger} \mid A_{K}=1, \bar{Y}_{k-1}^{\dagger}, Y_{0}, X, T, S=\mathbf{1}\right)
\end{aligned}
$$

- Posit a model for each component of the product.


## Modeling

$$
\begin{aligned}
& h\left(Y_{k}^{\dagger} \mid A_{K}=1, \bar{Y}_{k-1}^{\dagger}, Y_{0}, X, T=t, S=\mathbf{1}\right) \\
& \quad=h_{k, t}\left(Y_{k}^{\dagger}-\mu_{k, t}\left(\bar{Y}_{k-1}^{\dagger}, Y_{0}, X ; \boldsymbol{\alpha}_{k, t}\right)\right)
\end{aligned}
$$

- $\mu_{k, t}\left(\bar{Y}_{k-1}^{\dagger}, Y_{0}, X ; \boldsymbol{\alpha}_{k, t}\right)$ is a specified function
- $\boldsymbol{\alpha}_{k, t}$ is an unknown parameter vector
- $h_{k, t}$ is an unspecified time/treatment-specific density function.
- The parameter vectors $\boldsymbol{\alpha}_{k, t}$ can be estimated by minimizing the least squares objective function
$\sum_{i=1}^{n} I\left(T_{i}=t\right) A_{K, i}\left(\prod_{k=1}^{K} R_{k, i}\right)\left\{Y_{k, i}^{\dagger}-\mu_{k, t}\left(\bar{Y}_{k-1}^{\dagger}, Y_{0}, X ; \boldsymbol{\alpha}_{k, t}\right)\right\}^{2}$
- The density function $h_{k, t}$ can be estimated by kernel density estimation based on the residuals $\left\{Y_{k, i}^{\dagger}-\mu_{k, t}\left(\bar{Y}_{k-1, i}^{\dagger}, Y_{0, i}, X_{i} ; \widehat{\boldsymbol{\alpha}}_{k, t}\right): T_{i}=t, A_{K, i}=\right.$ $\left.1, R_{1, i}=\ldots, R_{K, i}=1, i=1, \ldots, n\right\}$
- $f\left(\bar{Y}_{K} \mid A_{K}=1, Y_{0}, X, T, S=\mathbf{1}\right)$ is estimated by

$$
\prod_{k=1}^{K} \widehat{h}_{k, t}\left(Y_{k}^{\dagger}-\mu_{k, t}\left(\bar{Y}_{k-1}^{\dagger}, Y_{0}, X ; \widehat{\boldsymbol{\alpha}}_{k, t}\right)\right)\left|\frac{d \phi\left(Y_{k}\right)}{d Y_{k}}\right| .
$$

## Imputation/Estimation

- For each individual $i$ alive at $t_{K}$ and who is in a stratum $s \neq 1$ and treatment $t$, impute the missing functional outcomes by drawing (using Metropolis-Hastings algorithm) from the density that is proportional to

$$
\left.\exp \left(\beta_{t} Z\right) \widehat{f( } Y_{m i s}^{(s)} \mid A_{K}=1, Y_{o b s}^{(s)}=Y_{o b s, i}, Y_{0}=Y_{0, i}, X=X_{i}, T=t, S=\mathbf{1}\right)
$$

- Draw $M$ copies of the missing functional outcomes to create $M$ complete datasets.
- For each complete dataset $m$, estimate $\theta$ by $\widehat{\theta}_{m}$.
- Overall estimator of $\theta$ is $\tilde{\theta}=\frac{1}{M} \sum_{m=1}^{M} \widehat{\theta}_{m}$.
- Confidence intervals can be constructed by non-parametric bootstrap


## Sampling steps

1. Set $j=0$. Choose arbitrary initial values for $Y_{\text {mis }}^{(s)}$, denoted by $Y_{m i s}^{(s, 0)}$. Let $Z_{i}^{(0)}$ be the primary functional endpoint with data $\left(Y_{o b s, i}, Y_{\text {mis }}^{(s, 0)}\right)$.
2. Set $j=j+1$
3. Generate $Y_{\text {mis }}^{(s)^{\prime}}$ from a (multivariate) Gaussian distribution with mean $Y_{\text {mis }}^{(s, j-1)}$ and variance $\Sigma$. Let $Z_{i}^{\prime}$ be the primary functional endpoint with data $\left(Y_{o b s, i}, Y_{\text {mis }}^{(s)^{\prime}}\right)$.

## Sampling steps

4. Calculate the acceptance ratio as

$$
\begin{aligned}
a & =\frac{\left.\exp \left\{\beta_{t} Z_{i}^{\prime}\right\} \widehat{f( } Y_{m i s}^{(s)^{\prime}} \mid A_{K}=1, Y_{o b s, i}, Y_{0, i}, X_{i}, T=t, S=\mathbf{1}\right)}{\exp \left\{\beta_{t} Z_{i}^{(j-1)}\right\} \widehat{f\left(Y_{m i s}^{(s, j)} \mid A_{K}=1, Y_{o b s, i}, Y_{0, i}, X_{i}, T=t, S=\mathbf{1}\right)}} \\
& =\frac{\left.\exp \left\{\beta_{t} Z_{i}^{\prime}\right\} \widehat{f\left(Y_{m i s}^{(s)^{\prime}}\right.}, Y_{o b s, i} \mid A_{K}=1, Y_{0, i}, X_{i}, T=t, S=\mathbf{1}\right)}{\left.\exp \left\{\beta_{t} Z_{i}^{(j-1)}\right\} \widehat{f\left(Y_{m i s}^{(s, j-1)}\right.}, Y_{o b s, i} \mid A_{K}=1, Y_{0, i}, X_{i}, T=t, S=\mathbf{1}\right)}
\end{aligned}
$$

## Sampling steps

5. Accept $Y_{\text {mis }}^{(s)^{\prime}}$ with probability $\min (1, a)$ and $Y_{\text {mis }}^{(s, j-1)}$ with probability $1-\min (1, a)$. Let $Y_{\text {mis }}^{(s, j)}$ be the accepted value.
6. Repeat Steps 2-5 until the Markov chain converges
7. Draw random samples from the set $\left\{Y_{m i s}^{\left(s, j_{0}\right)}, Y_{\text {mis }}^{(s, j o+1)}, \ldots\right\}$ as the imputed missing values, where $j_{0}$ corresponds to the number of burn-in

## Simulation scenarios

- Considered two post-baseline functional assessments at $t_{1}$ and $t_{2}$
- Scenario I
- Focused on evaluating the impact of survival and functional status among survivors
- Assume no missing data among survivors
- Scenario II
- Focused on evaluating the impact of missing data and the proposed sensitivity analysis strategy
- Assume no deaths


## Data generation

- Draw $Y_{0}$ from standard normal distribution.
- Given $T$ and $Y_{0}$, draw $L_{1}$ from an exponential distribution with mean $1 / \exp \left(\lambda_{T, 0}+\lambda_{T, 1} Y_{0}\right)$. If $L_{1}<t_{1}$, set $L=L_{1}$ and stop.
- Given $T$ and $Y_{0}$, draw $Y_{1}$ from a normal distribution with mean $\mu_{T}+\gamma_{T} Y_{0}$, and variance 1.
- Given $T$ and $\bar{Y}_{1}$, draw $L_{2}$ from an exponential distribution with mean $1 / \exp \left(\lambda_{T, 0}+\lambda_{T, 1} Y_{1}\right)$. If $L_{2}<t_{2}-t_{1}$, set $L=L_{2}+t_{1}$ and stop.
- Given $T$ and $\bar{Y}_{1}$, draw $Y_{2}$ from a normal distribution with mean $\mu_{T}+\gamma_{T} Y_{1}$ and variance 1.


## Data generation

- Given $T$ and $\bar{Y}_{2}$, draw $S$ from multinomial distribution with

$$
P\left[S=s \mid T, \bar{Y}_{2}\right]=\frac{\exp \left(\mu_{T, s}^{\prime}+\beta_{T} Z\right)}{1+\sum_{s^{\prime} \neq \mathbf{1}} \exp \left(\mu_{T, s^{\prime}}^{\prime}+\beta_{T} Z\right)}, \quad s \neq \mathbf{1}
$$

and

$$
P\left[S=\mathbf{1} \mid T, \bar{Y}_{2}\right]=\frac{1}{1+\sum_{s^{\prime} \neq \mathbf{1}} \exp \left(\mu_{T, s^{\prime}}^{\prime}+\beta_{T} Z\right)}
$$

## Scenario I results

| $\lambda_{1,1}$ | Death Rate |  | $\mu_{1}$ | True $\theta$ | Sample Size | Estimation |  | Rate |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $T=0$ | $T=1$ |  |  |  | $\theta$ | MSE* | Rej* | Cov* |
| 1.3 | 0.188 | 0.230 | 0.0 | -0.056 | 200 | -0.060 | 5.5 | 0.092 | 0.978 |
|  |  |  |  |  | 500 | -0.054 | 2.9 | 0.186 | 0.938 |
|  |  | 0.293 | 0.5 | 0.088 | 200 | 0.085 | 7.1 | 0.198 | 0.944 |
|  |  |  |  |  | 500 | 0.086 | 2.5 | 0.358 | 0.958 |
|  | 0.354 | 0.388 | 0.0 | -0.051 | 200 | -0.053 | 6.7 | 0.104 | 0.936 |
|  |  |  |  |  | 500 | -0.046 | 2.7 | 0.154 | 0.956 |
|  |  | 0.463 | 0.5 | 0.007 | 200 | 0.007 | 7.6 | 0.072 | 0.928 |
|  |  |  |  |  | 500 | 0.006 | 2.6 | 0.042 | 0.960 |
| 1.0 | 0.188 | 0.188 | 0.0 | -0.001 | 200 | 0.002 | 6.9 | 0.050 | 0.952 |
|  |  |  |  |  | 500 | 0.004 | 2.7 | 0.048 | 0.958 |
|  |  | 0.236 | 0.5 | 0.178 | 200 | 0.181 | 7.5 | 0.602 | 0.932 |
|  |  |  |  |  | 500 | 0.177 | 2.7 | 0.934 | 0.946 |
|  | 0.354 | 0.354 | 0.0 | 0.000 | 200 | -0.003 | 6.1 | 0.032 | 0.974 |
|  |  |  |  |  | 500 | 0.000 | 2.7 | 0.058 | 0.944 |
|  |  | 0.418 | 0.5 | 0.080 | 200 | 0.079 | 7.2 | 0.180 | 0.946 |
|  |  |  |  |  | 500 | 0.084 | 2.7 | 0.352 | 0.948 |
| 0.7 | 0.188 | 0.151 | 0.0 | 0.051 | 200 | 0.047 | 6.4 | 0.090 | 0.960 |
|  |  |  |  |  | 500 | 0.053 | 2.4 | 0.174 | 0.952 |
|  |  | 0.180 | 0.5 | 0.265 | 200 | 0.269 | 5.8 | 0.924 | 0.954 |
|  |  |  |  |  | 500 | 0.262 | 2.7 | 0.996 | 0.944 |
|  | 0.354 | 0.315 | 0.0 | 0.054 | 200 | 0.051 | 6.3 | 0.096 | 0.958 |
|  |  |  |  |  | 500 | 0.053 | 2.5 | 0.174 | 0.964 |
|  |  | 0.362 | 0.5 | 0.163 | 200 | 0.160 | 6.0 | 0.518 | 0.950 |
|  |  |  |  |  | 500 | 0.165 | 2.7 | 0.884 | 0.954 |

Table: Scenario I Simulation Study Results. MSE*: mean squared error $\times 1000$. Rej*: rejection rate for $H_{0}: \theta=0$. Cov*: bootstrap $95 \%$ confidence interval coverage rate. The Death Rates for $T=0$ are 0.188 or 0.354 corresponding to the study length ( $t_{2}$ ) of 0.2 and 0.5 , respectively.

## Scenario II results

| $\beta_{1}^{*}$ | Missing |  | $\begin{gathered} \hline \text { True } \\ \theta \end{gathered}$ | Sample Size | Estimation |  | Rate |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Rate* | $\mu_{1}$ |  |  | $\widehat{\theta}$ | MSE* | Rej* | Cov* |
| 0 | 0.21 | -0.25 | -0.186 | 200 | -0.049 | 26.8 | 0.090 | 0.640 |
|  |  |  |  | 500 | -0.045 | 23.5 | 0.146 | 0.268 |
|  | 0.15 | 0.00 | 0.000 | 200 | 0.104 | 18.4 | 0.236 | 0.780 |
|  |  |  |  | 500 | 0.110 | 15.1 | 0.516 | 0.476 |
|  | 0.10 | 0.25 | 0.186 | 200 | 0.275 | 14.4 | 0.906 | 0.810 |
|  |  |  |  | 500 | 0.271 | 9.5 | 1.000 | 0.614 |
| -2 | 0.21 | -0.25 | -0.186 | 200 | -0.192 | 7.1 | 0.612 | 0.952 |
|  |  |  |  | 500 | -0.189 | 2.9 | 0.928 | 0.950 |
|  | 0.15 | 0.00 | 0.000 | 200 | -0.014 | 7.6 | 0.054 | 0.952 |
|  |  |  |  | 500 | -0.011 | 3.1 | 0.050 | 0.952 |
|  | 0.10 | 0.25 | 0.186 | 200 | 0.180 | 7.5 | 0.572 | 0.950 |
|  |  |  |  | 500 | 0.178 | 2.7 | 0.928 | 0.948 |

Table: Scenario II Simulation Study Results. MSE*: mean squared error $\times 1000$. Rej*: rejection rate for $H_{0}: \theta=0$. Cov*: bootstrap $95 \%$ confidence interval coverage rate. $\beta_{1}^{*}$ : sensitivity parameter for $T=1$. Missing rate*: overall functional endpoint missing rate.

## HT-ANAM 302 Study

- Anamorelin is a drug developed for the treatment of cancer cachexia and anorexia.
- HT-ANAM 302 was a randomized, double-blind, placebo-controlled Phase III study designed to evaluate the efficacy of anamorelin in patients with advanced non-small cell lung cancer.
- Lean body mass (LBM) was scheduled to be measured at baseline $\left(Y_{0}\right), 6$ weeks $\left(Y_{1}\right)$ and 12 weeks $\left(Y_{2}\right)$
- Primary functional endpoint: $Z=\frac{\left(Y_{2}+Y_{1}\right)}{2}-Y_{0}$


## Death and missingness

|  | Placebo | Anamorelin |
| ---: | :---: | :---: |
|  | $n=157$ | $n=322$ |
| Died Prior to Wk 12 | $24(15.3 \%)$ | $54(16.8 \%)$ |
| Survivors with complete data | $93(59.2 \%)$ | $185(57.5 \%)$ |
| Survivors missing only Wk 6 | $3(1.9 \%)$ | $17(5.3 \%)$ |
| Survivors missing only Wk 12 | $17(10.8 \%)$ | $31(9.6 \%)$ |
| Survivors missing both Wks 6, 12 | $20(12.7 \%)$ | $35(10.9 \%)$ |

## Central Question

How should data from studies like HT-ANAM 302 be analyzed to evaluate the effect of treatment on the functional outcome?

## Missing pattern



## Completers LBM

Placebo



## Survival



## Baseline covariates

| Covariates | Levels |
| ---: | :--- |
| ECOG | $0:\{0,1\}, 1:\{2\}$ |
| AGE | $0: \leq 65,1:>65$ |
| GENDER | $0: M, 1: F$ |
| BMI | $0: \leq 18.5,1:>18.5$ |
| WEIGHT $^{\text {LOSS }}$ |  |
| YO | $0: \leq 10 \%, 1:>10 \%$ |
| Continuous |  |

${ }^{1}$ in prior 6 months

## Modeling

Specify $\mu_{k, t}\left(\bar{Y}_{k-1}, Y_{0}, X ; \boldsymbol{\alpha}_{k, t}\right)$ as follows:

$$
\begin{gathered}
\mu_{1, t}=\alpha_{1, t, 1}+\alpha_{1, t, 2} Y_{0}+\alpha_{1, t, 3} E C O G+\alpha_{1, t, 4} A G E \\
\quad+\alpha_{1, t, 5} G+\alpha_{1, t, 6} B M I+\alpha_{1, t, 7} W L \\
\mu_{2, t}=\alpha_{2, t, 1}+\alpha_{2, t, 2} Y_{0}+\alpha_{2, t, 3} E C O G+\alpha_{2, t, 4} A G E \\
\quad+\alpha_{2, t, 5} G+\alpha_{2, t, 6} B M I+\alpha_{2, t, 7} W L+\alpha_{2, t, 8} Y_{1}
\end{gathered}
$$

## Model fitting diagnosis



## Analysis under benchmark assumptions

- 10 imputed datasets generated
- 200 bootstrap samples

Table: Hypothesis testing

|  | $\hat{\theta}(95 \% \mathrm{CI})$ | p-value |
| :---: | :---: | :---: |
| HT-ANAM 302 Study | $0.30(0.19,0.40)$ | $<0.0001$ |

Table: Median

|  |  | $\widehat{p}_{50}(95 \% \mathrm{CI})$ |
| :--- | ---: | :---: |
| HT-ANAM 302 Study | Anamorelin | $0.67(0.45,0.89)$ |
|  | Placebo | $-0.92(-1.43,-0.28)$ |

## Cumulative plot

## Composite Endpoint



## Choice of sensitivity parameters



- Change in $E(Z)$ about 1.5 kg at $\beta_{T}=0.5$ and $\beta_{T}=-0.5$
- Set $\beta_{T}=\{-0.5,-0.4, \ldots, 0, \ldots, 0.5\}$


## Sensitivity analysis: Rank



## Sensitivity analysis: Median

Median


## Sensitivity analysis: Contour of p-values



## Conclusion

There is a significant difference between the Placebo and the Anamorelin arms in their composite endpoints of survival and average LBM change. The difference favors the Anamorelin arm.

## Discussion

- Method presumes that death and the functional outcome can be ordered in a scientifically meaningful way.
- Use mixed methods to confirm that ordering is consistent with the health preferences of patient population.
- Ranking scheme is similar to 'untied worst-rank score analysis" for missing data of Lachin (1999).
- The "worst-rank score analysis" ranks all the patients who died $\left(A_{K}=0\right)$ the same and is also commonly used.
- CCMV is a strong benchmark assumption.
- Assumed survival time is always known, need to extend methods to handle censoring.
- R package idem

