# MENDEL AND FISHER

# Mendel's Data

Model: seed shape governed by gene **A** with alleles $A, a$:

| Genotype | Phenotype |
|---|---|
| $AA$ | Round |
| $Aa$ | Round |
| $aa$ | Wrinkled |

Cross two inbred lines: $AA$ and $aa$. All offspring ($F_1$ generation) are $Aa$, and so have round seeds.

# $F_2$ generation

Self an $F_1$ plant: each allele it transmits is equally likely to be $A$ or $a$, and alleles are independent, so for $F_2$ generation:

$$\begin{aligned}
\Pr(AA) &= \Pr(A)\Pr(A) = 0.25 \\
\Pr(Aa) &= \Pr(A)\Pr(a) + \Pr(a)\Pr(A) = 0.5 \\
\Pr(aa) &= \Pr(a)\Pr(a) = 0.25
\end{aligned}$$

Probability that an $F_2$ seed (observed on $F_1$ parental plant) is round:

$$\begin{aligned}
\Pr(\text{Round}) &= \Pr(\text{Round}|AA)\Pr(\mathbf{AA}) \\
&\quad + \Pr(\text{Round}|Aa)\Pr(\mathbf{Aa}) \\
&\quad + \Pr(\text{Round}|aa)\Pr(\mathbf{aa}) \\
&= 1 \times \mathbf{0.25} + 1 \times \mathbf{0.5} + 0 \times \mathbf{0.25} \\
&= 0.75
\end{aligned}$$

# $F_2$ generation

What are the proportions of $AA$ and $Aa$ among $F_2$ plants with round seeds? From Bayes' Theorem the predicted probability of $AA$ genotype, if the seed is round, is

$$\begin{aligned}
\Pr(F_2 \text{ is } AA | F_2 \text{ is Round}) &= \frac{\Pr(F_2 \text{ is Round} | F_2, \text{ is } AA)\Pr(F_2 \text{ is } AA)}{\Pr(F_2 \text{ is round})} \\
&= \frac{1 \times \frac{1}{4}}{\frac{3}{4}} \\
&= \frac{1}{3}
\end{aligned}$$

# Seed Characters

As an experimental check on this last result, and therefore on Mendel's theory, Mendel selfed a round-seeded $F_2$ plant and noted the $F_3$ seed shape (observed on the $F_2$ parental plant).

If all the $F_3$ seeds are round, the $F_2$ must have been $AA$. If some $F_3$ seed are round and some are wrinkled, the $F_2$ must have been $Aa$. Possible to observe many $F_3$ seeds for an $F_2$ parental plant, so no doubt that all seeds were round. Data supported theory: one-third of $F_2$ plants gave only round seeds and so must have had genotype $AA$.

# Plant Characters

Model for stem length is

| Genotype | Phenotype |
|---|---|
| $GG$ | Long |
| $Gg$ | Long |
| $gg$ | Short |

To check this model it is necessary to grow the $F_3$ seed to observe the $F_3$ stem length.

# $F_2$ Plant Character

Mendel grew only 10 $F_3$ seeds per $F_2$ parent. If all 10 seeds gave long stems, he concluded they were all $GG$, and $F_2$ parent was $GG$. This could be wrong. The probability of a $Gg$ $F_2$ plant giving 10 long-stemmed $F_3$ offspring ($GG$ or $Gg$), and therefore wrongly declared to be homozygous $GG$ is $(3/4)^{10} = 0.0563$.

# Fisher's 1936 Criticism

The probability that a long-stemmed $F_2$ plant is declared to be homozygous (event $V$) is

$$\begin{aligned} \Pr(V) &= \Pr(V|U)\Pr(U) + \Pr(V|\bar{U})\Pr(\bar{U}) \\ &= 1 \times (1/3) + 0.0563 \times (2/3) \\ &= 0.3709 \\ &\neq 1/3 \end{aligned}$$

where $U$ is the event that a long-stemmed $F_2$ is actually homozygous and $\bar{U}$ is the event that it is actually heterozygous.

Fisher claimed Mendel's data closer to the 0.3333 probability appropriate for seed shape than to the correct 0.3709 value. Mendel's experiments were "a carefully planned demonstration of his conclusions."

Fisher RA. 1936. Has Mendel's work been rediscovered? Ann. Sci. 1: 115-137.

# Weldon's 1902 Doubts

In Biometrika, Weldon said:

"Here are seven determinations of a frequency which is said to obey the law of Chance. Only one determination has a deviation from the hypothetical frequency greater than the probable error of the determination, and one has a deviation sensible equal to the probable error; so that a discrepancy between the hypothesis and the observations which is equal to or greater than the probable error occurs twice out of seven times, and deviations much greater than the probable error do not occur at all. These results then accord so remarkably with Mendel's summary of them that if they were repeated a second time, under similar conditions and on a similar scale, the chance that the agreement between observation and hypothesis would be worse than that actually obtained is about 16 to 1."

"Run Mendel's experiments again at the same scale, Weldon reckoned, and the chance of getting worse results is 16 to 1."
Radick, Science 350:159-160, 2015.

# Edwards' 1986 Criticism

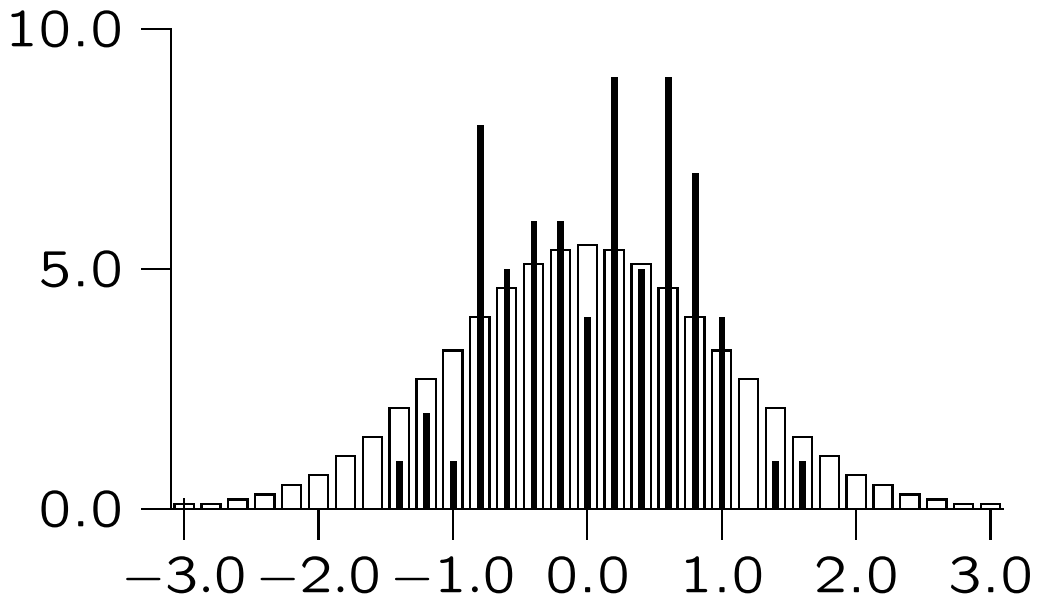
Mendel had 69 comparisons where the expected ratios were correct. Each set of data can be tested with a chi-square test:

| | | Category 1 | Category 2 | Total |
|---|---|---|---|---|
| Observed | (o) | a | n-a | n |
| Expected | (e) | b | n-b | n |

$$\begin{aligned} X^2 &= \frac{(a-b)^2}{b} + \frac{[(n-a)-(n-b)]^2}{(n-b)} \\ &= \frac{n(a-b)^2}{b(n-b)} \end{aligned}$$

# Edwards' Criticism

If the hypothesis giving the expected values is true, the $X^2$ values follow a chi-square distribution, and the $X$ values follow a normal distribution. Edwards claimed Mendel's values were too small – not as many large values as would be expected by chance.




Edwards AWF. 1986. Are Mendel's results really too close? Biological Reviews 61:295-312.

# 2018 paper

"According to Fisher (1959), if the null hypothesis is rejected, 'The force with which such a conclusion is supported is that of the simple disjunction: Either an exceptionally rare chance has occurred, or the theory of random distribution is not true' (p. 39). Fisher's theory does not permit one to say which of the two possibilities is the case, nor to give a probability for it. Furthermore, if significance is not achieved, nothing can be concluded. In order for the probability distribution that forms the basis of a chi-square test to be valid, the hypothesis to be tested must be declared before the data are examined.

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# 2018 paper

Viewed in this light, there are several gaps between Fisher's calculations and his conclusion. Fisher is rejecting the multinomial null hypothesis if the chi-square is too small, which would be legitimate if the hypothesis test were declared before Weldon pointed the way, or if Fisher routinely used a two-tailed chi-square test. Neither is the case. And one still has Fisher's disjunction to contend with. Nonetheless, Fisher is a superb data-analyst, and we should not be interpreted as challenging his conclusion."

Kadane JB, Wang Z. 2018. Sums of possibly associated multivariate indicator functions; the Conway-Maxwell-Multinomial distribution. Brazilian Journal of Probability and Statistics 32:583-596.

# Other Recent Discussions

Franklin A, Edwards AWF, Fairbanks DJ, Hartl DL, Seidenfeld T. 2008. "Ending the Mendel-Fisher Controversy." University of Pittsburgh Press, Pittsburgh.

Novitski E. 2004. On Fisher's criticism of Mendel's results with the garden pea. Genetics 155:133-1136.

Smith MU, Gericke NM. 2015. Mendel in the modern classroom. Science and Education 24:151-172.

Radick G. 2015. Beyond the "Mendel-Fisher controversy." Science 350:159-160.

Weeden NF. 2016. Are Mendel's Data Reliable? The Perspective of a Pea Geneticist. Journal of Heredity 107:635-646. "Mendel's article is probably best regarded as his attempt to present his model in a simple and convincing format with a minimum of additional details that might obscure his message."

# R.A. Fisher, 2020, US.

The Committee of Presidents of Statistical Societies (COPSS) is retiring the R.A. Fisher Award and Lecture, effective immediately.

We take this action to advance a more just, equitable, diverse, and inclusive statistical community. ... We recognize Fisher's fundamental contributions in establishing statistics as a scientific discipline. We heard the voices of those who argued for further deliberation before finalizing a decision. We have confidence that we will all work together to achieve our common goal of a fair, just, and equitable society and profession.

https://community.amstat.org/copss/home June 23, 2020

# R.A. Fisher, 2020, UK.

"A Cambridge college is to take down a window commemorating a eugenicist that was installed 30 years ago, after pressure from anti-racism activists.

Sir Ronald Fisher, the founding chairman of the University of Cambridge eugenics society, is memorialised in the stained glass window but Gonville and Caius College council said it recognised it caused 'broad offence' and that they should no longer honour him.

In a statement, the college said it would take it down subject to listed building consent after discussions this week. Recently, activists wrote 'eugenics is genocide  Fisher must fall' on its Gate of Honour."

The Guardian, June 27, 2020

# Fisher and Eugenics

"In 1911 Fisher became founding Chairman of the University of Cambridge Eugenics Society ...He saw eugenics as addressing pressing social and scientific issues that encompassed and drove his interest in both genetics and statistics. "

https://en.wikipedia.org/wiki/Ronald_Fisher

"Available scientific knowledge provides a firm basis for believing that the groups of mankind differ in their innate capacity for intellectual and emotional development, seeing that such groups do differ undoubtedly in a very large number of their genes."

Fisher RA. p 56 in https://unesdoc.unesco.org/ark:/48223/pf0000073351