Answers to most exercises in notes by Tom Britton

Ex. 2.

R_0	τ^*
1.5	0.583
3	0.94
6	0.997

Ex 3.

R_0	τ^* (= P(outbreak) for R-F)	P(outbreak) when I~Exp
1.5	0.583	0.33
3	0.94	0.67
6	0.997	0.83

Ex 4+5

R_0	τ^*	Overall fraction $(=(1-v)\tau^*)$
1.5	0	0
3	0.583	0.29
6	0.94	0.47

Ex 6 Diseases with $R_0 \le 1$ have no name...

Ex 7.

Week	$I(t)$
0	1
1	$e^{2.8} \approx 16$
2	$e^{5.6} \approx 250$
3	$e^{8.4} \approx 4000$

Answers... cont'd

Ex. 8

R_0	V_c
1.5	33%
3	67%
6	82%

Ex. 9

R_0	$\tilde{s}$
1.5	0.67
3	0.33
6	0.18

Ex 10 $\tilde{i} = \frac{1/52}{75} = 1/3900 \approx 0.00026$

Ex 11. $n \cdot \tilde{i} \approx 60$ for Iceland and $n\tilde{i} \approx 15000$ for UK

Ex 12. H will die out in Iceland but persist in UK

Ex 13 Conf int for p: $\hat{p} \pm \lambda_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}$

$\Rightarrow 0.27 \pm 1.96 \cdot \sqrt{0.27 \cdot 0.73/100} = 0.27 \pm 0.09$

Ex 14 $\hat{R}_0 = \frac{-\ln 0.8}{0.2} \approx 1.12$

Answers ... cont'd

Ex. 15 $\tilde{\tau} = 0.4$ $1-r = 0.5$

$$\Rightarrow \hat{R}_0 = \frac{-\ln(0.6)}{0.5 \cdot 0.4} \approx 2.55$$

Ex. 16 $\hat{V}_c = \frac{1}{0.9}\left(1 - \frac{0.2}{-\ln 0.6}\right) = 68\%$

Ex. 17 $\hat{\rho} = \frac{\ln(121/7)}{3-1} = 1.42$ per week

Ex. 18 $\hat{V}_c = \frac{1}{0.95}\left(1 - \frac{1}{15}\right) \approx 99\%$

If $E = 0.9 \Rightarrow V_c = \ldots = 104\% \Rightarrow$ vaccine cannot alone prevent an outbreak

Ex. 22 α_i = infectivity of type i

β_j = susceptibility of type j

Ex. 23. $\alpha_1 = 1,\ \alpha_2 = 2,\ \gamma_1 = 1,\ \gamma_2 = 2,\ \pi_1 = \pi_2 = 0.5$

$$\Rightarrow R_0 = \sum_1^2 \alpha_i \gamma_i \pi_i = 2.5$$

Surprizing?

$M = \begin{pmatrix} 0.5 & 1 \\ 1 & 2 \end{pmatrix}$

Type 1 infects on average $0.5 + 1 = 1.5$

" 2 " $1 + 2 = 3$

Ex 23 continued

The mean of these ($\pi_1 = \pi_2 = 0.5$) equals $2.25 < 2.5$

Explanation: R_0 is bigger than 2.25 because type 2 will be over-represented among infected during early stage of the outbreak, and they infect more individuals

Ex. 24 $\quad p = 0.25 \quad E(D) = 3 \quad R_0 = p\left(E(D) + \frac{V(D) - E(D)}{E(D)}\right)$

$\sqrt{V(D)}$	$V(D)$	R_0	$V_c = 1 - \frac{1}{R_0}$
0	0	0.5	0
1	1	0.9	0
3	9	1.25	0.2
10	100	≈ 9	≈ 0.88

Ex. 25 $\quad \hat{R}_0 = \frac{-\ln(1-\tilde{\tau})}{\tilde{\tau}} = \frac{-\ln\left(\frac{412}{651}\right)}{239/651} \approx 1.25$

$$\hat{V}_c = 1 - \frac{1}{\hat{R}_0} \approx 0.20$$

$$s.e.(\hat{R}_0) = \sqrt{\frac{1 + C_v^2(1-\tilde{\tau})\hat{R}_0^2}{\tilde{\tau}(1-\tilde{\tau}) \cdot n}} = \begin{cases} 0.08 & C_v = 0 \\ 0.11 & C_v = 1 \end{cases}$$

$$s.e(\hat{V}_c) = \sqrt{\frac{1 + C_v^2(1-\tilde{\tau})\hat{R}_0^2}{\hat{R}_0^4\,\tilde{\tau}(1-\tilde{\tau})\,n}} = \begin{cases} 0.05 & C_v = 0 \\ 0.07 & C_v = 1 \end{cases}$$

Ex. 26: Multiply by $\sqrt{10}$ for s.e.