Solutions / Answers to Tom Britton's Exercises

Exercises L1

1. Ex 1. $P(Z=3) = P(i \to 3 \to 0) + P(i \to 2 \to 1 \to 0) + P(i \to 1 \to 2 \to 0) + P(i \to 1 \to 1 \to 1 \to 0)$

E.g. $P(i \to 1 \to 2 \to 0)$

$$= \binom{n-1}{1} p^1 (1-p)^{n-2} \cdot \binom{n-2}{2} p^2 (1-p)^{n-4} \cdot \binom{n-4}{0} \left(1-(1-p)^2\right)^0 \left((1-p)^2\right)^{n-4}$$

2. Ex 2

R_0	τ^*_0
0.8	0
1.5	0.583
3	≈0.940
6	≈0.997

These are also outbreak prob

τ^* positive sol of

$$1-\tau = e^{-R_0 \tau}$$

Ex 3 G.E.M

R_0	τ^*	P(outbreak)
1.5	0.583	$\frac{1}{3}$ 0.33
3	≈0.98	$\frac{2}{3}$ 0.66
6	≈0.999	$\frac{5}{6}$ 0.83

Ex 4 5

R_0	τ^*	over all frac infected
1.5	0	0
3	0.583	0.39
6	0.98	0.49

Exercises L2

6: Others unknown

7 Exercise

ρ solution to $1 = \int_0^\infty e^{-\rho t} \mu(dt)$

Week(t)	I(t)
0	1
1	$e^{2.8} \approx 16$
2	$e^{5.6} \approx 250$
3	$e^{8.4} \approx 4000$

8 Exercise

R_0	V_c
1.5	0.33
3	0.68
6	0.82

9 Exercise

R_0	$\hat{s}$
1.5	0.67
3	0.33
6	0.17

~~Conversely. Suppose $\hat{s} = \frac{\text{av. age at inf}}{\text{av life-length}} \approx \frac{5\text{ yr}}{75\text{ yr}} = \frac{1}{15}$ known~~

~~$\Rightarrow \hat{s} \cdot R_0 = 1 \Rightarrow R_0 = \frac{1}{\hat{s}} \approx 15$!~~

10 Exercise $\hat{i} = \frac{\frac{1}{50}}{75} = \frac{1}{3750}$

11 Exercise $n\hat{i} \approx 60$ for Iceland $n\hat{i} \approx 15\,000$ for England

12. Die out in Iceland & persist in England

Exercise L3

13 **Exercise** Conf int for p: $\hat{p} \pm \overset{\lambda_{\alpha/2}}{\cancel{1.96}} \cdot \sqrt{\hat{p}(1-\hat{p})/n}$

$$= 0.27 \pm 1.96 \cdot \sqrt{\frac{0.27 \cdot 0.73}{10}} = 0.27 \pm 0.09$$

14 **Exercise** $$\hat{R}_0 = \frac{-\ln(1-\tilde{\tau})}{\tilde{\tau}} = \frac{-\ln 0.8}{0.2} \approx$$

15 **Exercise** $\tilde{\tau} = 0.4$ $\quad 1-r = 0.5$

$$\Rightarrow \hat{R}_0 = \frac{-\ln(0.6)}{0.\cancel{5}\,0.4} \approx 2.55$$

16 **Exercise** $$\hat{V}_c = \frac{1}{0.9}\left(1 - \frac{(1-r)\,\tilde{\tau}}{-\ln(1-\tilde{\tau})}\right) = \frac{1}{0.9}\left(1 - \frac{0.2}{-\ln(0.6)}\right) = 68\%$$

17 **Exercise** $$\hat{\rho} = \frac{\ln(121/7)}{2}$$

18 **Exercise** $$\hat{V}_c = \frac{1}{E}\left(1 - \frac{1}{15}\right) = \frac{1}{0.9}\left(1 - \frac{1}{15}\right) \approx \frac{10 \cdot 14}{9 \cdot 15} = 104\%$$

Ändra till $E = 0.95$ $\Rightarrow$ $V_c \approx 99\%$

Exercises L4

21 $\frac{\beta_{ij}}{n} \cdot \pi_j \cdot n\, \nu = \beta_{ij} \pi_j \nu$

24 $p = 0.25$ $E(D) = 3$ $= p\left(E(D) + \frac{V(D) - E(D)}{E(D)}\right)$

$\sqrt{V(D)}$	$V(D)$	R_0	$V_c = 1 - \frac{1}{R_0}$
0	0	0.5	0
1	1	0.9	0
3	9	1.25	0.2
10	100	≈ 9	0.88

22. α_i = infectivity of i-individuals

β_j = susceptibility of j-individuals

23. $\alpha_1 = 1$ $\alpha_2 = 2$ $\beta_1 = 1$ $\beta_2 = 2$

$\Rightarrow R_0 = \sum \alpha_i \beta_i \nu \pi_i = \frac{1+4}{2} = 2.5$

$M = \begin{pmatrix} \frac{1}{2} & 1 \\ 1 & 2 \end{pmatrix}$ Type 1 ind infect 1.5, Type 2 ind infect 3 } On average 2.25

L8

$n = 651 \quad z = 239 \quad \tilde{\tau} = \frac{239}{651}$

25 Exercise $\hat{R}_0 = \frac{-\ln(1-\tilde{\tau})}{\tilde{\tau}} = \frac{-\ln\left(\frac{412}{651}\right)}{239/651} = \ldots 1.25$ $\quad 1-\tilde{\tau} = \frac{412}{651}$

$$\begin{array}{r} 651 \\ -239 \\ \hline 412 \end{array}$$

$$\hat{V}_c = 1 - \frac{1}{\hat{R}_0} = 0.20$$

$$\text{s.e. } \hat{R}_0 = \sqrt{\frac{1 + r^2(1-\tilde{\tau})\hat{R}_0^2}{\tilde{\tau}(1-\tilde{\tau})\, n}} = \begin{cases} 0.08 & r=0 \\ 0.11 & r=1 \end{cases}$$

$$\text{s.e.}(\hat{V}_c) = \sqrt{\frac{1 + r^2(1-\tilde{\tau})\hat{R}_0^2}{\hat{R}_0^4\, \tilde{\tau}(1-\tilde{\tau})\, n}} = \begin{cases} 0.05 & r=0 \\ 0.07 & r=0 \end{cases}$$

26 Exercise Same point estimate

std reduced by factor $\frac{1}{\sqrt{10}}$