L2, Important properties of epidemics and endemic situations

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July, 2022

The basic reproduction number

Recall: $R_0 =$ expected number individuals a typical infected person infects when everyone is susceptible

R_0 depends both on disease (infectious agent) and on community!!

$R_0 < 1$ or $R_0 > 1$ makes a very big difference!

Next page: R_0 for some diseases (and communities and time periods), Anderson and May, 1991

R_0 for some diseases, communities and time periods (Anderson & May, 1991)

Table 4.1 Estimated values of the basic reproductive rate, R_0, for various infections (data from Anderson (1982*b*), Anderson and May (1982*d*, 1985*c*, 1988), Anderson *et al.* (1988), Nokes and Anderson (1988)).

Infection	Geographical location	Time period	R_0
Measles	Cirencester, England	1947–50	13–14
	England and Wales	1950–68	16–18
	Kansas, USA	1918–21	5–6
	Ontario, Canada	1912–13	11–12
	Willesden, England	1912–13	11–12
	Ghana	1960–8	14–15
	Eastern Nigeria	1960–8	16–17
Pertussis	England and Wales	1944–78	16–18
	Maryland, USA	1943	16–17
	Ontario, Canada	1912–13	10–11
Chicken pox	Maryland, USA	1913–17	7–8
	New Jersey, USA	1912–21	7–8
	Baltimore, USA	1943	10–11
	England and Wales	1944–68	10–12
Diphtheria	New York, USA	1918–19	4–5
	Maryland, USA	1908–17	4–5
Scarlet fever	Maryland, USA	1908–17	7–8
	New York, USA	1918–19	5–6
	Pennsylvania, USA	1910–16	6–7
Mumps	Baltimore, USA	1943	7–8
	England and Wales	1960–80	11–14
	Netherlands	1970–80	11–14
Rubella	England and Wales	1960–70	6–7
	West Germany	1970–7	6–7
	Czechoslovakia	1970–7	8–9
	Poland	1970–7	11–12
	Gambia	1976	15–16
Poliomyelitis	USA	1955	5–6
	Netherlands	1960	6–7
Human Immunodeficiency Virus (Type I)	England and Wales (male homosexuals)	1981–5	2–5
	Nairobi, Kenya (female prostitutes)	1981–5	11–12

Exercise 6: Why is $R_0 > 1$ for all diseases above?

Initial growth rate ρ

Exponential growth rate due to "branching" behavior

$I(t) \approx e^{\rho t}$

ρ depends more on specific model assumptions (contact rate, latency period, infectious period, ...)

R_0 and ρ (unfortunately) not too related

R_0 more important

ρ easier to estimate *during* an outbreak

Exercise 7: Suppose the exponential growth rate ρ equals $\rho = 2.8$ (per week) and that there is one index case week 0. Compute the expected incidence ($\approx I(t)$) after 1, 2 and 3 weeks.

Time events and terminology

Infection-related events in time

Infection time: calendar time when someone gets infected (unobserved)

Report time: calendar time an infected is reported a case (observed for reported cases)

Onset of symptoms: calendar time when an infected notices symptoms (often observed for reported cases)

Incubation period: relative time between getting infected and having symptoms (unobserved)

Latent period: relative time between getting infected and becomen infectious (SEIR models) (unobserved)

Generation time and serial interval: relates times between infector and infectee – more next lecture

Modelling vaccination

Why is modelling of disease spread important?

Modelling vaccination

Why is modelling of disease spread important?

Increase understanding and *prevention* (e.g. **vaccination**)

Suppose that a fraction v are vaccinated prior to outbreak

Assume first a perfect vaccine (100% immunity)

$\Longrightarrow$ a fraction v are initially immune (discussed in previous lecture)

R_v is the reproduction number after a fraction v has been vaccinated

$\Longrightarrow R_v = R_0(1-v)$

$R_v < 1$ equivalent to $R_0(1-v) < 1$ equivalent to $v > 1 - 1/R_0$

Modelling vaccination cont'd

So, if $v > 1 - 1/R_0$ there will be no major outbreak: "Herd immunity"

$v_c = 1 - 1/R_0$ is called the *critical vaccination coverage*

Exercise 8: Compute v_c for a disease having $R_0 = 1.5$, 3 and 6

On next page are estimates of v_c for some diseases

v_c for some diseases (Anderson & May, 1991)

Modelling vaccination cont'd

If vaccine is not perfect but relative risk of getting infected from an infectious contact for vaccinees is $1 - E$, $0 < E \leq 1$ (E for "efficacy", later to be called VE_S), then

$$v_c = \frac{1}{E}\left(1 - \frac{1}{R_0}\right)$$

For a highly infectious disease (R_0 large) and a not so effective vaccine (E not too close to 1) v_c might exceed 1. This means vaccination alone cannot prevent an outbreak!

More on modelling and inference of vaccine effects later in course

Endemic diseases

When interest is on long-term situation (as opposed to short term outbreaks) the assumption of a fixed population must be relaxed

Consider an SIR disease in a population where individuals die and new are born. Assume:

- SIR disease (life long immunity)
- population at "equilibrium" (in terms of size and incidence)
- disease endemic (constantly present, no big fluctuations)
- $\tilde{s}$, $\tilde{i}$ and $\tilde{r}$ denote the average fractions susceptible, infectious and removed
- $R_0 =$ average number of infections caused by one individual – if everyone was susceptible!

Think of childhood diseases (e.g. chicken-pox)

Endemic diseases, expression for $\tilde{s}$

When disease is in endemic equilibrium each infected individual on average infects exactly 1 new person!

Given R_0 and $\tilde{s}$ an infected individual infects on average $R_0\tilde{s}$ new individuals

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$\Longrightarrow R_0\tilde{s} = 1$!!

$\tilde{s} = \frac{1}{R_0}$

$\tilde{s} =$ average fraction susceptible $= \frac{\text{average age at infection}}{\text{average life-length}}$

Exercise 9 Suppose $R_0 = 1.5$, 3 and 6 respectively, compute $\tilde{s}$.

Endemic diseases, expression for $\tilde{i}$

If ι is the average length of infectious period and ℓ average life-length, then ι/ℓ is the average time of the life an individual is infectious

Since population/disease in equilibrium this is also the population fraction of infectives

$$\tilde{i} = \frac{\iota}{\ell}$$

Exercises

Exercise 10 Consider an endemic disease with one week infectious period and a population with 75 years expected life-length. Compute the average fraction infective $\tilde{i}$.

Exercise 11 Consider the disease in the previous exercise and consider the Icelandic population ($n = 250\ 000$). What is the average *number* of infectives? How about England ($n = 60\ 000\ 000$)?

Exercise 12 What do you think will happen with the disease in the two countries (remember that if the number of infectives drops to 0 the disease goes extinct - until it is "re-imported")?