

## L2, Important properties of epidemics and endemic situations

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# The basic reproduction number

Recall:  $R_0$  = expected number individuals a typical infected person infects when everyone is susceptible

$R_0$  depends both on disease (infectious agent) and on community!!

$R_0 < 1$  or  $R_0 > 1$  makes a very big difference!

Next page:  $R_0$  for some diseases (and communities and time periods), Anderson and May, 1991

# $R_0$ for some diseases, communities and time periods (Anderson & May, 1991)

70 *Microparasites*

**Table 4.1** Estimated values of the basic reproductive rate,  $R_0$ , for various infections (data from Anderson (1982*b*), Anderson and May (1982*d*, 1985*c*, 1988), Anderson *et al.* (1988), Nokes and Anderson (1988)).

Infection	Geographical location	Time period	$R_0$
Measles	Cirencester, England	1947–50	13–14
	England and Wales	1950–68	16–18
	Kansas, USA	1918–21	5–6
	Ontario, Canada	1912–13	11–12
	Willesden, England	1912–13	11–12
	Ghana	1960–8	14–15
Pertussis	Eastern Nigeria	1960–8	16–17
	England and Wales	1944–78	16–18
	Maryland, USA	1943	16–17
	Ontario, Canada	1912–13	10–11
Chicken pox	Maryland, USA	1913–17	7–8
	New Jersey, USA	1912–21	7–8
	Baltimore, USA	1943	10–11
Diphtheria	England and Wales	1944–68	10–12
	New York, USA	1918–19	4–5
	Maryland, USA	1908–17	4–5
Scarlet fever	Maryland, USA	1908–17	7–8
	New York, USA	1918–19	5–6
	Pennsylvania, USA	1910–16	6–7
Mumps	Baltimore, USA	1943	7–8
	England and Wales	1960–80	11–14
	Netherlands	1970–80	11–14
Rubella	England and Wales	1960–70	6–7
	West Germany	1970–7	6–7
	Czechoslovakia	1970–7	8–9
	Poland	1970–7	11–12
Poliomyelitis	Gambia	1976	15–16
	USA	1955	5–6
	Netherlands	1960	6–7
Human Immunodeficiency Virus (Type I)	England and Wales (male homosexuals)	1981–5	2–5
	Nairobi, Kenya (female prostitutes)	1981–5	11–12

**Exercise 6:** Why is  $R_0 > 1$  for all diseases above?

## Initial growth rate $\rho$

Exponential growth rate due to "branching" behavior

$$I(t) \approx e^{\rho t}$$

$\rho$  depends more on specific model assumptions (contact rate, latency period, infectious period, ...)

$R_0$  and  $\rho$  (unfortunately) not too related

$R_0$  more important

$\rho$  easier to estimate *during* an outbreak

**Exercise 7:** Suppose the exponential growth rate  $\rho$  equals  $\rho = 2.8$  (per week) and that there is one index case week 0. Compute the expected incidence ( $\approx I(t)$ ) after 1, 2 and 3 weeks.

## Time events and terminology

Infection-related events in time

**Infection time:** calendar time when someone gets infected  
(unobserved)

**Report time:** calendar time an infected is reported a case  
(observed for reported cases)

**Onset of symptoms:** calendar time when an infected notices  
symptoms (often observed for reported cases)

**Incubation period:** relative time between getting infected and  
having symptoms (unobserved)

**Latent period:** relative time between getting infected and  
becomen infectious (SEIR models) (unobserved)

**Generation time and serial interval:** relates times between  
infector and infectee – more next lecture

# Modelling vaccination

Why is modelling of disease spread important?

## Modelling vaccination

Why is modelling of disease spread important?

Increase understanding and *prevention* (e.g. **vaccination**)

Suppose that a fraction  $v$  are vaccinated prior to outbreak

Assume first a perfect vaccine (100% immunity)

$\implies$  a fraction  $v$  are initially immune (discussed in previous lecture)

$R_V$  is the reproduction number after a fraction  $v$  has been vaccinated

$$\implies R_V = R_0(1 - v)$$

$R_V < 1$  equivalent to  $R_0(1 - v) < 1$  equivalent to  $v > 1 - 1/R_0$



## Modelling vaccination cont'd

So, if  $v > 1 - 1/R_0$  there will be no major outbreak: "Herd immunity"

$v_c = 1 - 1/R_0$  is called the *critical vaccination coverage*

**Exercise 8:** Compute  $v_c$  for a disease having  $R_0 = 1.5, 3$  and  $6$

On next page are estimates of  $v_c$  for some diseases

# $v_c$ for some diseases (Anderson & May, 1991)

**Fig. 5.1.** The dependence of the critical level of vaccination coverage required to halt transmission,  $p_c$ , on the basic reproductive rate  $R_0$ , or, equivalently, on the average age at infection,  $A$  (see eqns (5.2) and (5.3)).

**Table 5.1** Approximate estimates of the vaccination coverage (the degree of herd immunity) required to eradicate a variety of viral, bacterial, and protozoan infections in developed and developing countries (eqn (5.2) in the main text)

Infectious disease	Critical proportions ( $p_c$ ) of the population to be immunized for eradication
Malaria ( <i>P. falciparum</i> in a hyperendemic region)	99%
Measles	90–95%
Whooping cough (pertussis)	90–95%
Fifth disease (human parvovirus infection)	90–95%
Chicken pox	85–90%
Mumps	85–90%
Rubella	82–87%
Poliomyelitis	82–87%
Diphtheria	82–87%
Scarlet fever	82–87%
Smallpox	70–80%

## Modelling vaccination cont'd

If vaccine is not perfect but relative risk of getting infected from an infectious contact for vaccinees is  $1 - E$ ,  $0 < E \leq 1$  ( $E$  for "efficacy", later to be called  $VE_S$ ), then

$$v_c = \frac{1}{E} \left( 1 - \frac{1}{R_0} \right)$$

For a highly infectious disease ( $R_0$  large) and a not so effective vaccine ( $E$  not too close to 1)  $v_c$  might exceed 1. This means vaccination alone cannot prevent an outbreak!

More on modelling and inference of vaccine effects later in course

## Endemic diseases

When interest is on long-term situation (as opposed to short term outbreaks) the assumption of a fixed population must be relaxed

Consider an SIR disease in a population where individuals die and new are born. Assume:

- SIR disease (life long immunity)
- population at "equilibrium" (in terms of size and incidence)
- disease endemic (constantly present, no big fluctuations)
- $\tilde{s}$ ,  $\tilde{i}$  and  $\tilde{r}$  denote the average fractions susceptible, infectious and removed
- $R_0$  = average number of infections caused by one individual – if everyone was susceptible!

Think of childhood diseases (e.g. chicken-pox)

## Endemic diseases, expression for $\tilde{s}$

When disease is in endemic equilibrium each infected individual on average infects exactly 1 new person!

Given  $R_0$  and  $\tilde{s}$  an infected individual infects on average  $R_0\tilde{s}$  new individuals

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$$\implies R_0\tilde{s} = 1 !!$$

$$\tilde{s} = \frac{1}{R_0}$$

$$\tilde{s} = \text{average fraction susceptible} = \frac{\text{average age at infection}}{\text{average life-length}}$$

**Exercise 9** Suppose  $R_0 = 1.5, 3$  and  $6$  respectively, compute  $\tilde{s}$ .

## Endemic diseases, expression for $\tilde{i}$

If  $\iota$  is the average length of infectious period and  $\ell$  average life-length, then  $\iota/\ell$  is the average time of the life an individual is infectious

Since population/disease in equilibrium this is also the population fraction of infectives

$$\tilde{i} = \frac{\iota}{\ell}$$

## Exercises

**Exercise 10** Consider an endemic disease with one week infectious period and a population with 75 years expected life-length. Compute the average fraction infective  $\tilde{i}$ .

**Exercise 11** Consider the disease in the previous exercise and consider the Icelandic population ( $n = 250\,000$ ). What is the average *number* of infectives? How about England ( $n = 60\,000\,000$ )?

**Exercise 12** What do you think will happen with the disease in the two countries (remember that if the number of infectives drops to 0 the disease goes extinct - until it is "re-imported")?