

Modern Statistical Learning Methods for Observational Biomedical Data

Lab 0: Identification of an average treatment effect

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MODULE 8

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Contents of this Lab

- 1 Computing an ATE using the G-computation and IPTW formulas by hand

The G-computation and IPTW identification formulas

For the population described in the table below, calculate $E(Y | A = a)$ and $E[Y(a)]$ corresponding to treatment ($a = 1$) and control ($a = 0$) using:

- (i) the G-computation formula;
- (ii) the IPTW formula.

(Green and orange values provide counts of individuals with $Y = 1$ and $Y = 0$, resp.)

	$W = 0$	$W = 1$	$W = 2$	
$A = 1$	20 + 12	49 + 11	89 + 7	
$A = 0$	94 + 34	30 + 30	6 + 18	

The G-computation and IPTW identification formulas

	$W = 0$	$W = 1$	$W = 2$	
$A = 1$	$20 + 12$	$49 + 11$	$89 + 7$	188
$A = 0$	$94 + 34$	$30 + 30$	$6 + 18$	212
	160	120	120	400

The G-computation and IPTW identification formulas

$$\bar{Q}_n(1, 0) = \frac{12}{32} = 0.375 \quad \Bigg| \quad \bar{Q}_n(1, 1) = \frac{11}{60} = 0.183 \quad \Bigg| \quad \bar{Q}_n(1, 2) = \frac{7}{96} = 0.073$$

$$\bar{Q}_n(0, 0) = \frac{34}{128} = 0.266 \quad \Bigg| \quad \bar{Q}_n(0, 1) = \frac{30}{60} = 0.5 \quad \Bigg| \quad \bar{Q}_n(0, 2) = \frac{18}{24} = 0.75$$

$$g_n(0) = \frac{32}{160} = 0.2 \quad \Bigg| \quad g_n(1) = \frac{60}{120} = 0.5 \quad \Bigg| \quad g_n(2) = \frac{96}{120} = 0.8$$

$$P_n(W = 0) = \frac{160}{400} = 0.4 \quad \Bigg| \quad P_n(W = 1) = \frac{120}{400} = 0.3 \quad \Bigg| \quad P_n(W = 2) = \frac{120}{400} = 0.3$$

The G-computation and IPTW identification formulas

$$\begin{aligned}
 \gamma_{n,G} &= [\bar{Q}_n(1,0)P_n(W=0) + \bar{Q}_n(1,1)P_n(W=1) + \bar{Q}_n(1,2)P_n(W=2)] \\
 &\quad - [\bar{Q}_n(0,0)P_n(W=0) + \bar{Q}_n(0,1)P_n(W=1) + \bar{Q}_n(0,2)P_n(W=2)] \\
 &= \left[\frac{12}{32} \cdot \frac{160}{400} + \frac{11}{60} \cdot \frac{120}{400} + \frac{7}{96} \cdot \frac{120}{400} \right] - \left[\frac{34}{128} \cdot \frac{160}{400} + \frac{30}{60} \cdot \frac{120}{400} + \frac{18}{24} \cdot \frac{120}{400} \right] \\
 &= 0.227 - 0.481 = -0.254
 \end{aligned}$$

$$\begin{aligned}
 \gamma_{n,IPTW} &= \left[\frac{P_n(Y=1, A=1, W=0)}{g_n(0)} + \frac{P_n(Y=1, A=1, W=1)}{g_n(1)} + \frac{P_n(Y=1, A=1, W=2)}{g_n(2)} \right] \\
 &\quad - \left[\frac{P_n(Y=1, A=0, W=0)}{1-g_n(0)} + \frac{P_n(Y=1, A=0, W=1)}{1-g_n(1)} + \frac{P_n(Y=1, A=0, W=2)}{1-g_n(2)} \right] \\
 &= \left[\frac{12}{400} / \frac{32}{160} + \frac{11}{400} / \frac{60}{120} + \frac{7}{400} / \frac{96}{120} \right] \\
 &\quad - \left[\frac{34}{400} / \left(1 - \frac{32}{160}\right) + \frac{30}{400} / \left(1 - \frac{60}{120}\right) + \frac{18}{400} / \left(1 - \frac{96}{120}\right) \right] \\
 &= 0.227 - 0.481 = -0.254
 \end{aligned}$$