Overview/reminder of basic concepts in statistics and genetics

Random variables, expected values and (co)variance

A discrete random variable can assume only a countable number of values

Probability mass function:

$$p(x) = P(X = x)$$

Expected value:

$$\mu = E(X) = \sum xp(x)$$

As a function of random variable:

$$E[h(X)] = \sum h(x)p(x)$$

Variance:

$$Var(X) = E[(X - \mu)^2]$$

Random variables, expected values and (co)variance

A discrete random variable can assume only a countable number of values

allele *x*:

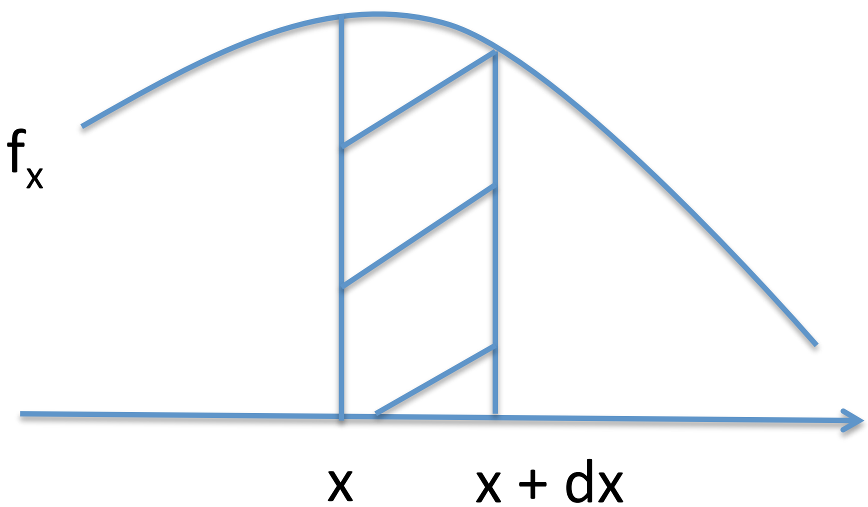
$$p(x) = \left\{ \begin{array}{cc} p & x = 1 \\ 1 - p & x = 0 \end{array} \right\}$$

$$E(X) = 0(1 - p) + 1(p) = p$$

$$Var(X) = p - p^2 = p(1 - p)$$

Random variables, expected values and (co)variance

A <u>continuous</u> random variable can be any value within a range



probability of being in shaded area

$$= f_X(x)dx$$

the interval should contribute

$$= xf_X(x)dx$$

the expected value and variance

$$E(X) = \mu_X = \int_{-\infty}^{\infty} xf_X(x)\,dx \qquad Var(X) = E((X - \mu_x)^2)$$

Random variables, expected values and (co)variance

Covariance

Let X and Y be a pair of continuous random variables, with respective means μ_x and μ_y. The expected value of $(X - \mu_x)(Y - \mu_y)$ is called the covariance between X and Y.

$$Cov(X,Y) = E\left[\left(X - \mu_x\right)\left(Y - \mu_y\right)\right]$$

If the random variables X and Y are independent, then the covariance between them is 0. However, the converse is not true.

Summary (co)variance rules

$$Var(x) = E[x - E(x)]^2$$

$$Var(cx) = c^2 Var(x)$$

$$Var(x + y) = Var(x) + Var(y) + 2Cov(x, y)$$

$$Var(x + c) = Var(x)$$

$$Cov(x, y) = E[(x - E(x)(y - E(y))]$$

$$Cov(cx, y) = cCov(x, y)$$

$$Cov(x, y + z) = Cov(x, y) + Cov(x, z)$$

Bayes' Theorem

Identify people who are liable to suffer from a genetic disease later in life.

1 in 1000 people are a carrier of the disease

No test is perfect - probability that a carrier tests negative is 1%

- probability that a non-carrier tests positive is 5%

A = the event "the patient is a carrier"

B = the event "the test result is positive"

Hence: $P(A) = 0.001$; $P(A') = 0.999$; $P(B|A) = 0.99$; $P(B|A') = 0.05$

A patient has a positive result. **Q**: What is the probability that the patient is a carrier?

Answer

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

$$= \frac{0.99 * 0.001}{(0.99 * 0.001) + (0.05 * 0.999)} = 0.0194$$

Hardy-Weinberg equilibrium

Mathematical relation between **allele** frequencies and the **genotype** frequencies is:

AA: p^2 Aa: $2pq$ aa: q^2

Allele	A	a
Frequency	p	q

Allele	Frequency		
A	p	AA p^2	Aa pq
a	q	aA qp	aa q^2

HWE and SNPs

If SNP genotypes are coded X = 0, 1 and 2 (alleles) and the allele frequency is p, then:

$$E(X) = (1-p)^2*0 + 2p(1-p)*1 + p^2*2 = 2p$$

$$var(X) = (1-p)^2*(0-2p)^2 + 2p(1-p)*(1-2p)^2 + p^2*(2-2p)^2 = 2p(1-p)$$