

MODULE 6: INTRODUCTION TO SURVIVAL ANALYSIS

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OVERVIEW

- Session 1
 - Introductory examples
 - The survival function
 - Survival Distributions
 - Mean and Median survival time
- Session 2
 - Censored data
 - Risk sets
 - Censoring Assumptions
 - Kaplan-Meier Estimator and CI
 - Median and CI
- Session 3
 - Two-group comparisons: logrank test
 - Trend and heterogeneity tests for more than two groups
- Session 4
 - Introduction to Cox regression

OVERVIEW – MODULE 9

Module 9: Survival analysis in Clinical Trials

- Quick review of basics
- Estimating survival after Cox model fit
- More two-sample tests
 - Weighted logrank
 - Additional tests based on functionals and metrics
- Adjustment, precision and post-randomization variables
- Power
- Choice of outcome
- Information accrual in sequential monitoring

OVERVIEW – MODULE 13

Module 13: Survival analysis for Observational Data

- More complicated Cox models
 - Adjustment
 - Interaction
- Competing Risks
- Choice of time variable
- Left Entry/Truncation
- Immortal time bias
- Index event bias
- Time-dependent covariates

MODULE 6
INTRODUCTION TO SURVIVAL ANALYSIS

SESSION 1:
SURVIVAL DATA: EXAMPLES

PRELIMINARIES

- No prior knowledge of survival analysis techniques assumed
- Familiarity with standard one- and two-sample statistical methods (estimation and testing) is assumed
- Emphasis on application rather than mathematical details
- Examples

SESSIONS/BREAKS

- 8:30 – 10:00
 - Break until 10:30
- 10:30 – 12:00
 - Break until 1:30
- 1:30 – 3:00
 - Break until 3:30
- 3:30 – 5:00

WHAT IS SURVIVAL ANALYSIS ABOUT?

- Studies the occurrence of an event over time
 - Time from randomization to death (cancer RCT)
 - Time from acceptance into a heart transplant program to death
 - Time from randomization to diagnosis of Alzheimer's Disease in a prevention trial
 - Time from randomization to ovarian cancer death in a randomized screening trial
 - Time from birth to removal of supplementary oxygen therapy
 - Time from first VTE diagnosis to recurrent VTE

YOUR EXAMPLES

WHAT IS SURVIVAL ANALYSIS ABOUT?

- Explores factors that are thought to influence the chance that the event occurs
 - Treatment
 - Age
 - Gender
 - Body Mass Index
 - Diet

 - Etc.

EXAMPLE 1

- Levamisole and Fluorouracil for adjuvant therapy of resected colon carcinoma
Moertel et al., 1990, 1995
- 1296 patients, enrolled 1 – 5 weeks after surgery
- Stage B₂ or C
- 3 unblinded treatment groups in stage C (2:1:1 ratio)
 - Observation only
 - Levamisole (oral, 1yr)
 - Levamisole (oral, 1yr) + fluorouracil (intravenous 1yr)

[Moertel CG, Fleming TR, Macdonald JS, et al. \(1990\) NEJM: 322\(6\):352–358.](#)

[Moertel CG. et al \(1995\). Annals of Internal Medicine: 122\(5\):321.](#)

EXAMPLE 1

- Randomization
 - Dynamic method based on accrued:
 - For B_2 , extent of invasion, time since surgery
 - For C, extent of invasion, time since surgery, number of lymph nodes involved

EXAMPLE 1

- Statistical analysis
 - Survival primary outcome (recurrence secondary)
 - Kaplan-Meier survival curves
 - Log-rank statistic
 - Cox proportional-hazards model for all multivariable analysis
 - Backward regression, maximal partial-likelihood estimate statistic
 - O'Brien-Fleming boundary for sequential monitoring; stopped early for stage C

EXAMPLE 1

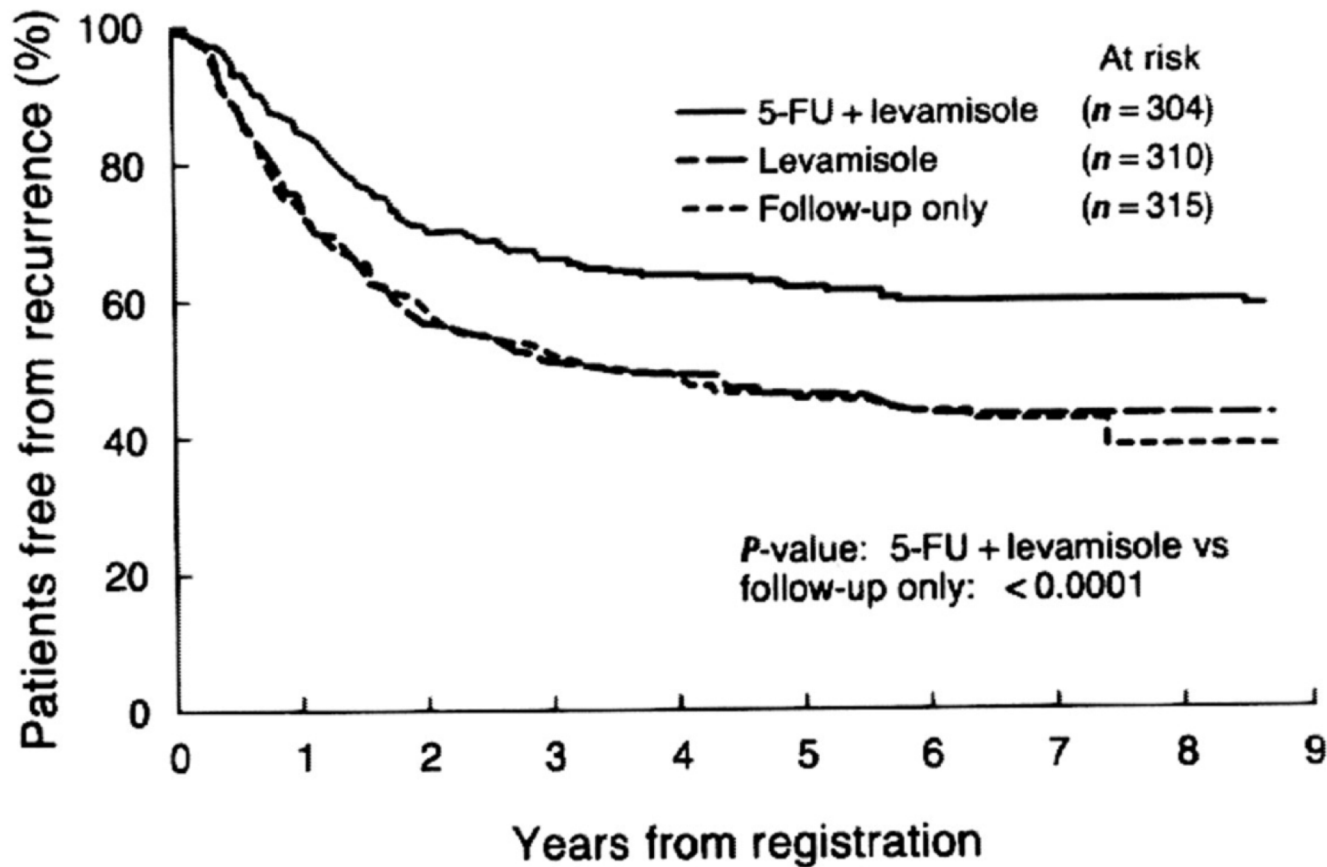


Figure 1: Recurrence-free interval according to treatment arm. Patients who died without recurrence have been censored. 5-FU = fluorouracil.

From: Fluorouracil plus Levamisole as Effective Adjuvant Therapy after Resection of Stage III Colon Carcinoma: A Final Report

Ann Intern Med. 1995;122(5):321-326. doi:10.7326/0003-4819-122-5-199503010-00001

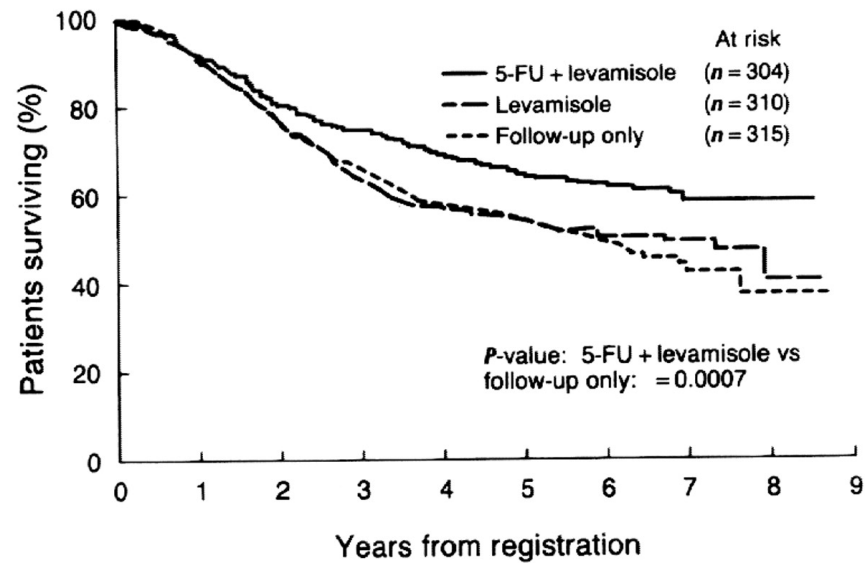


Figure Legend:

Survival according to treatment arm. 5-FU = fluorouracil.

EXAMPLE 1

- **Results** (stage C) after 2nd interim analysis
- Fluorouracil + Levamisole reduced the
 - Recurrence rate by 41% (95% CI 23% - 54%) (p<0.0001)
 - Death rate by 33% (95% CI 10% - 50%) (p<0.006)
- Levamisole reduced the
 - Recurrence rate by 2%
 - Death rate by 6%
- Toxicity was mild (with few exceptions)
- Patient compliance excellent

EXAMPLE 1

- R survival package data “colon”
 - 929 eligible stage C patients (971 randomized – 42 ineligible)
 - Treatment groups (rx)
 - Sex, age
 - Obstruction of colon by tumor (obstruct)
 - Perforation of colon (perfor)
 - Adherence to nearby organs (adhere)
 - Number of lymph nodes with detectable cancer (nodes)
 - Days until event or censoring (time)
 - Censoring status (status)

EXAMPLE 1

- Multivariable analysis:
 - Proportional hazards model
 - “we kept the variable of treatment in the model and used backward regression for other covariates”
 - Other covariates ($P < 0.01$)
 - Depth of primary tumor invasion,
 - Invasion of adjacent structures
 - Regional implants
 - Number of metastatic lymph nodes
 - Histological differentiation
 - Preoperative carcinoembryonic antigen level

EXAMPLE 1

- Multivariable results: “After correction for the influence of prognostic factors through the use of a proportional hazards model, patients receiving fluorouracil plus levamisole were again found to have a significant survival advantage when compared with patients assigned to observation only; they had a 33% reduction in mortality rate (95% CI, 16% to 47%; $P = 0.0007$). Therapy with levamisole alone showed essentially no effect (6% reduction in death rate; $P = 0.57$.”

Moertel et al (1995)

EXAMPLE 2 – ALZHEIMER'S

- Petersen et al. 2005, NEJM
- Subjects with amnesic subtype of mild cognitive impairment
- Adaptive randomization based on MMSE score, age, Apo ϵ 4 genotype
- Three arms: Vitamin E, Donepezil, and Placebo
- Primary outcome: Time from randomization to possible or probable AD diagnosis
- Length of double-blind treatment: 3 years

[Petersen RC, Thomas RG, Grundman M. et al. \(2005\) NEJM. 352\(23\):2379–2388.](#)

EXAMPLE 2 – ALZHEIMER'S

- Primary analysis: Cox regression adjusted for randomization influencing variables MMSE score, age and Apo E genotype
- 769 enrolled: 253 donepezil, 257 vitamin E, 259 placebo
- 230 dropped out: 92 donepezil, 74 vitamin E, 66 placebo
 - Treatment related toxicity: GI complaints, muscle aches, insomnia
- Dropout was observed to be related to MMSE score

EXAMPLE 2 – ALZHEIMER'S

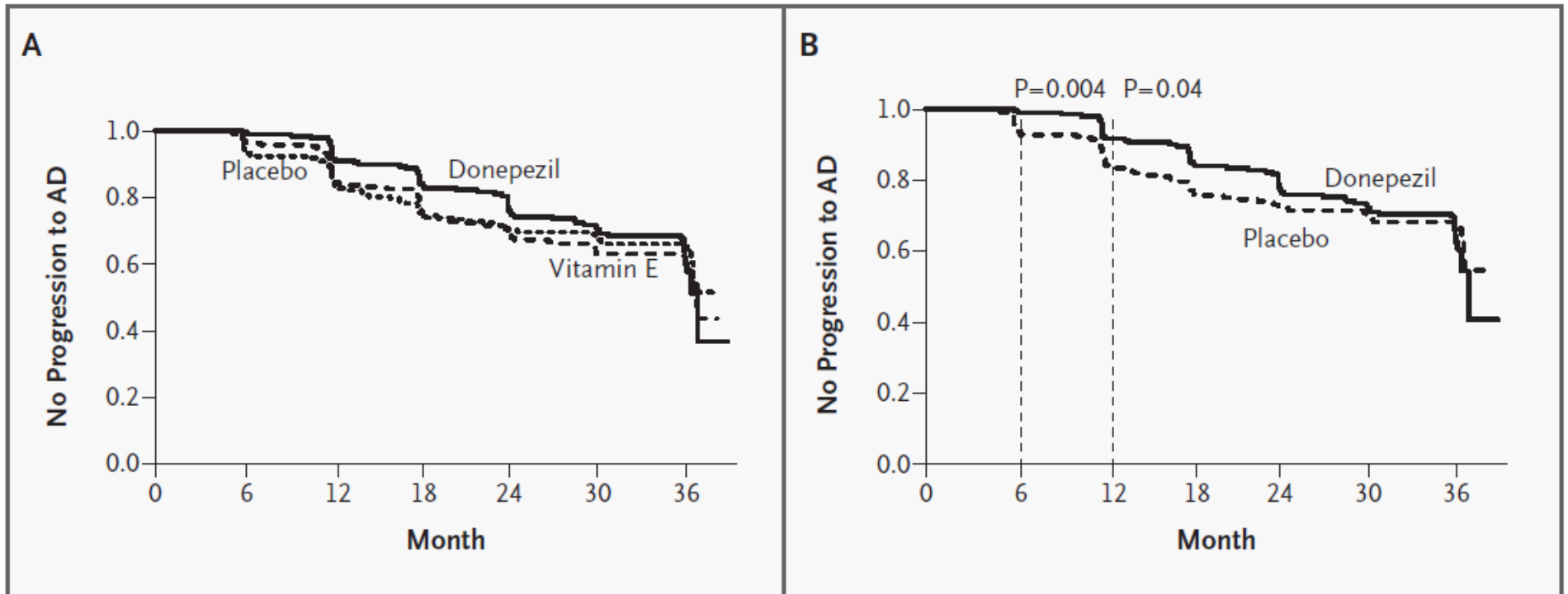
- 212 developed possible or probable AD
- “There were no significant differences ... during the three years of treatment”
- Vitamin E vs Placebo
 - Hazard Ratio 1.02 (95% CI, 0.74, 1.41), p-value 0.91
- Donepezil vs Placebo
 - Hazard Ratio 0.80 (95% CI, 0.57, 1.13), p-value 0.42

EXAMPLE 2 – ALZHEIMER'S

- Pre-specified analyses at 6 month intervals
 - Donepezil vs Placebo significantly reduced likelihood of progression to AD during the first 12 months (p-value 0.04)
 - Finding supported by secondary outcome measures
 - Subgroup ≥ 1 apolipoprotein E $\epsilon 4$ alleles significantly reduced likelihood of progression to AD over 3 years
 - Vitamin E vs Placebo: no significant differences
 - Vitamin E vs Placebo: also no significance for above subgroup
- Simulations assuming informative treatment-related dropout did not change primary conclusions

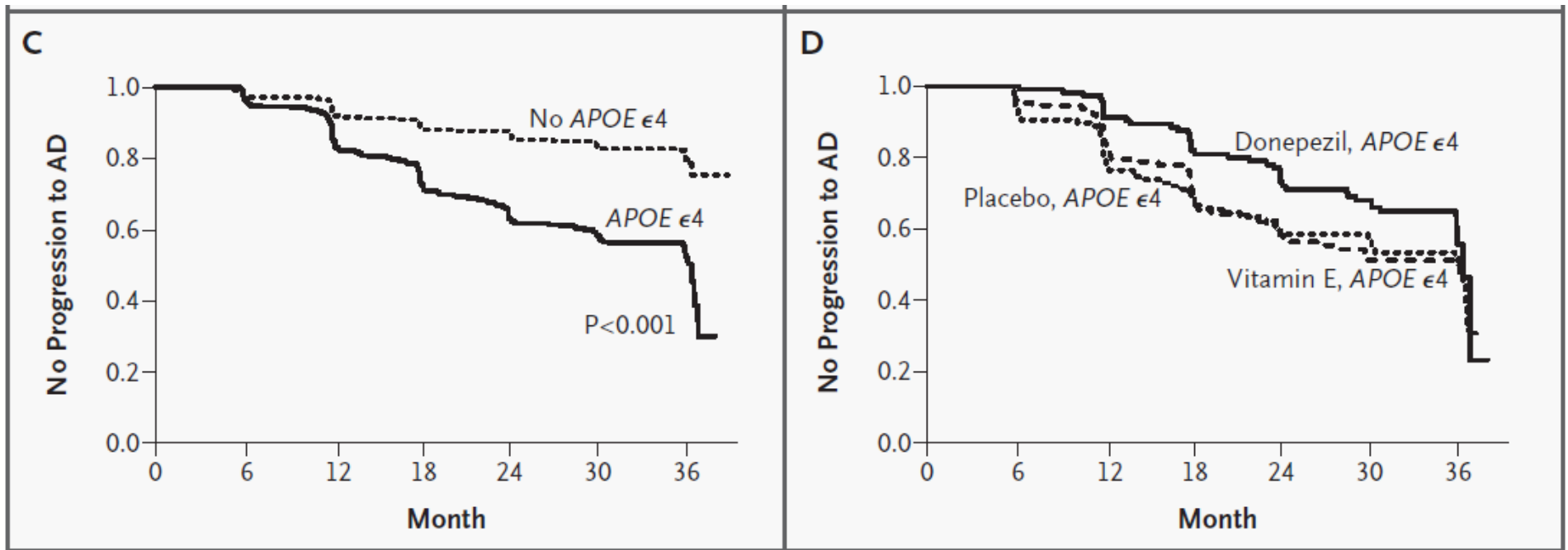
EXAMPLE 2 – RESULTS

- Overall and at 6 and 12 months



EXAMPLE 2 – RESULTS

- APOE $\epsilon 4$ results



EDITORIAL

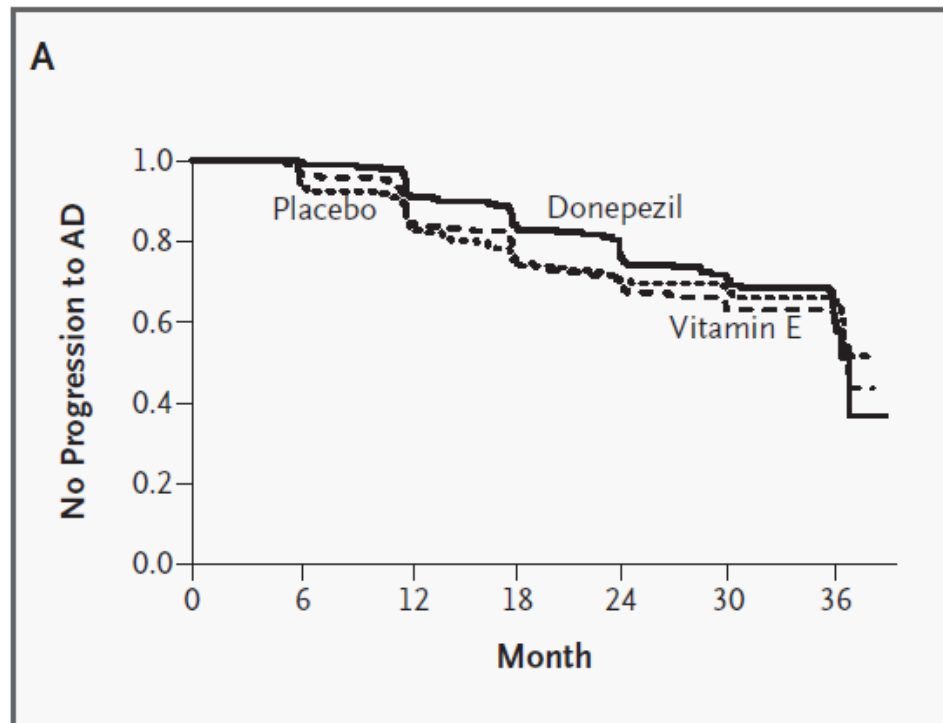
- “long-awaited results”
- Donepezil standard therapy for AD
- “Implications Enormous”
 - “The clear-cut negative findings for vitamin E, which is widely used despite the dearth of evidence of its efficacy, are especially noteworthy.”
 - Findings for donepezil “much less clear”

EDITORIAL COMMENTS

- “rate of progression ... somewhat lower in the treatment group during the first year of the study”
- “by two years, even this small effect had worn off”
- Possible explanation: “Reduced statistical power later in the study as the number of subjects at risk declined owing to death, withdrawal and development of AD
- Secondary analyses suggest... benefits wore off

EXAMPLE 2 – RESULTS

- Interesting steps.....



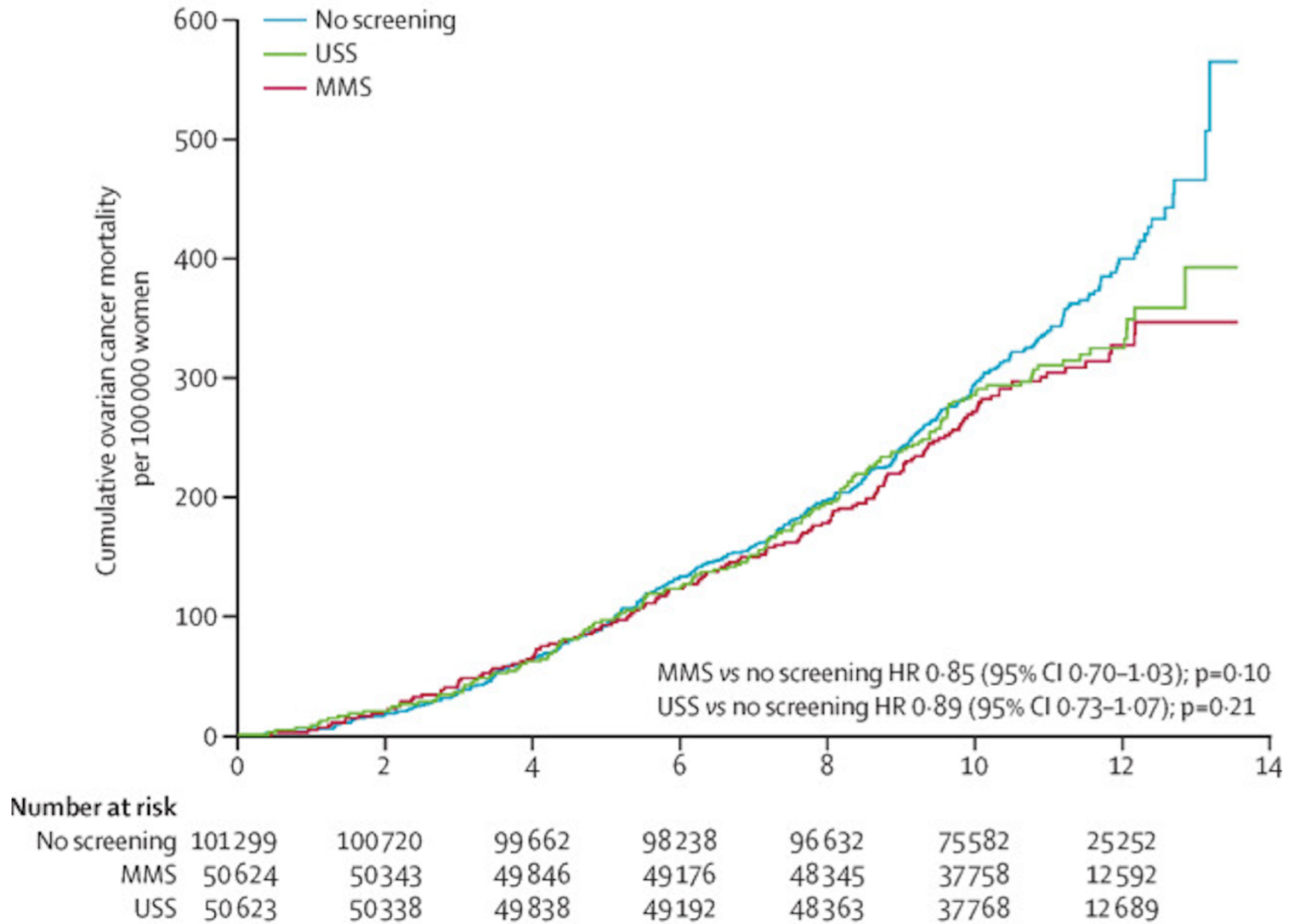
SCREENING TRIAL

- 202,546 women 50-72 years of age, England, Wales, Northern Ireland
 - Randomized to one of three arms in 1:1:2 ratio between June 1, 2001 and Oct 21, 2005.
 - Annual multimodal screening (serum CA 125 + algorithm)
 - Annual transvaginal ultrasound
 - No screening
 - Screening ended Dec 31, 2011.
 - Not blinded
 - Primary outcome: death from ovarian cancer (by end of 2014)
- [Jacobs IJ, Menon U, Ryan A, et al. \(2016\) The Lancet. 387\(10022\):945–956.](#)

OVARIAN CANCER SCREENING TRIAL

- Primary analysis: Cox regression (proportional hazards)
 - MMS vs. no screening: Mortality reduction = $(1 - \text{HR})100 = 15\%$ (95% CI: -1% – 33%) P = .10
 - USS vs. no screening: Mortality reduction = $(1 - \text{HR}) 100 = 11\%$ (95% CI: -7% - 27%) P = .21

OVARIAN CANCER SCREENING TRIAL



OVARIAN CANCER SCREENING TRIAL

- Why the delayed difference?

OVARIAN CANCER SCREENING TRIAL

- Secondary analyses conducted motivated by late separation of survival curves
- Post-hoc Weighted* logrank test:
 - MMS mortality reduction = 22% (3-38%) P = .023
 - USS mortality reduction = 20% (0 – 35%) P = .049

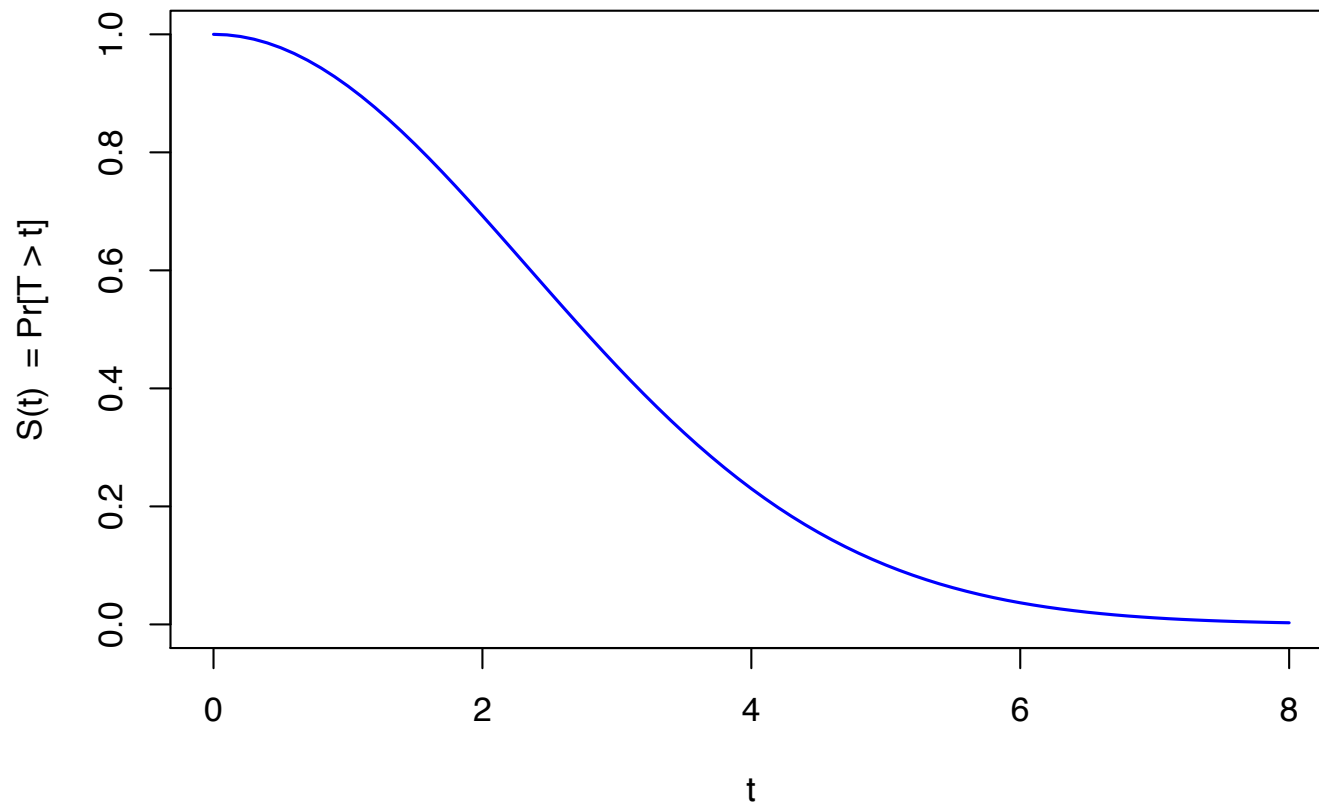
* by pooled cumulative mortality

SURVIVAL DATA AND FUNCTION

- Original applications in biometry were to survival times in cancer clinical trials
- Many other applications in biometry: eg. disease onset ages
- Interest centers not only on average or median survival time but also on probability of surviving beyond 2 years, 5 years, 10 years, etc.
- Best described with the entire survival function $S(t)$.
 - For $T =$ a subject's survival time, $S(t) = P[T > t]$.
 - Characterizes the entire distribution of survival times T .
 - Gives useful information for each t .

SURVIVAL FUNCTION

Survival Function

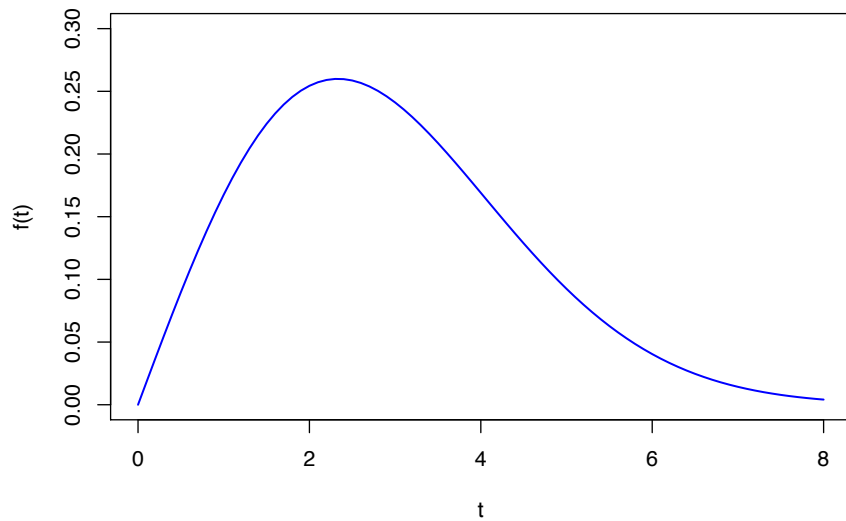


SURVIVAL DISTRIBUTION

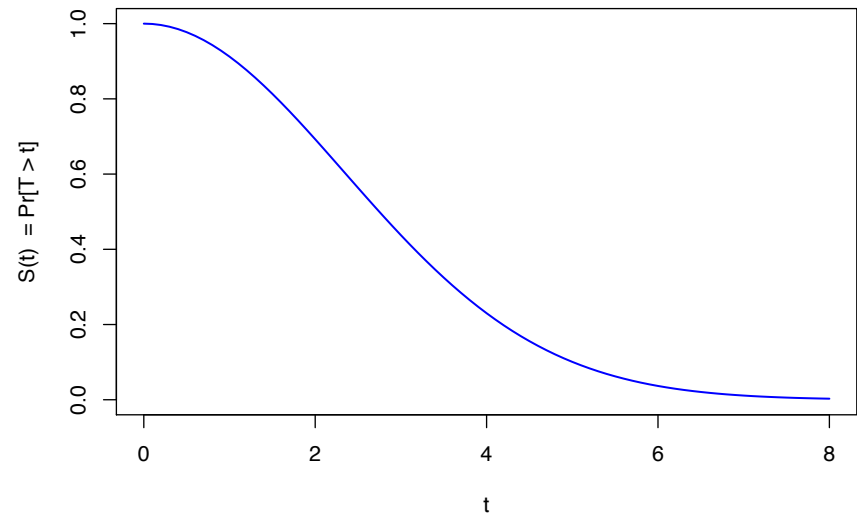
- Continuous probability distribution of times T
- Only non-negative T 's are possible: $\Pr(T < 0) = 0$
- Density function $f(t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \Pr(t \leq T < t + \Delta t)$
- Area under the $f(t)$ curve between two points is the probability T is between the two points.

DENSITY AND SURVIVAL FUNCTIONS

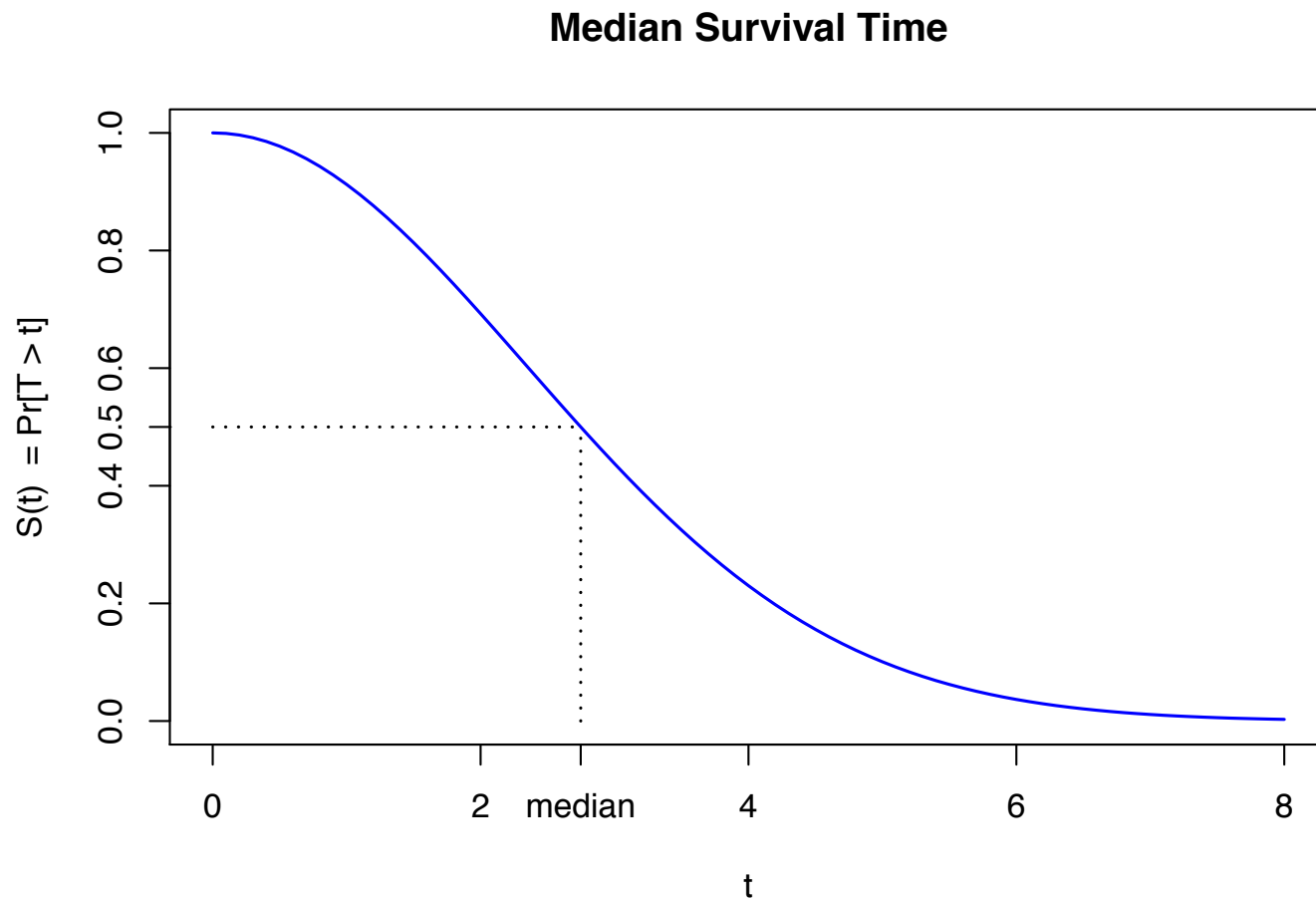
Density Function



Survival Function

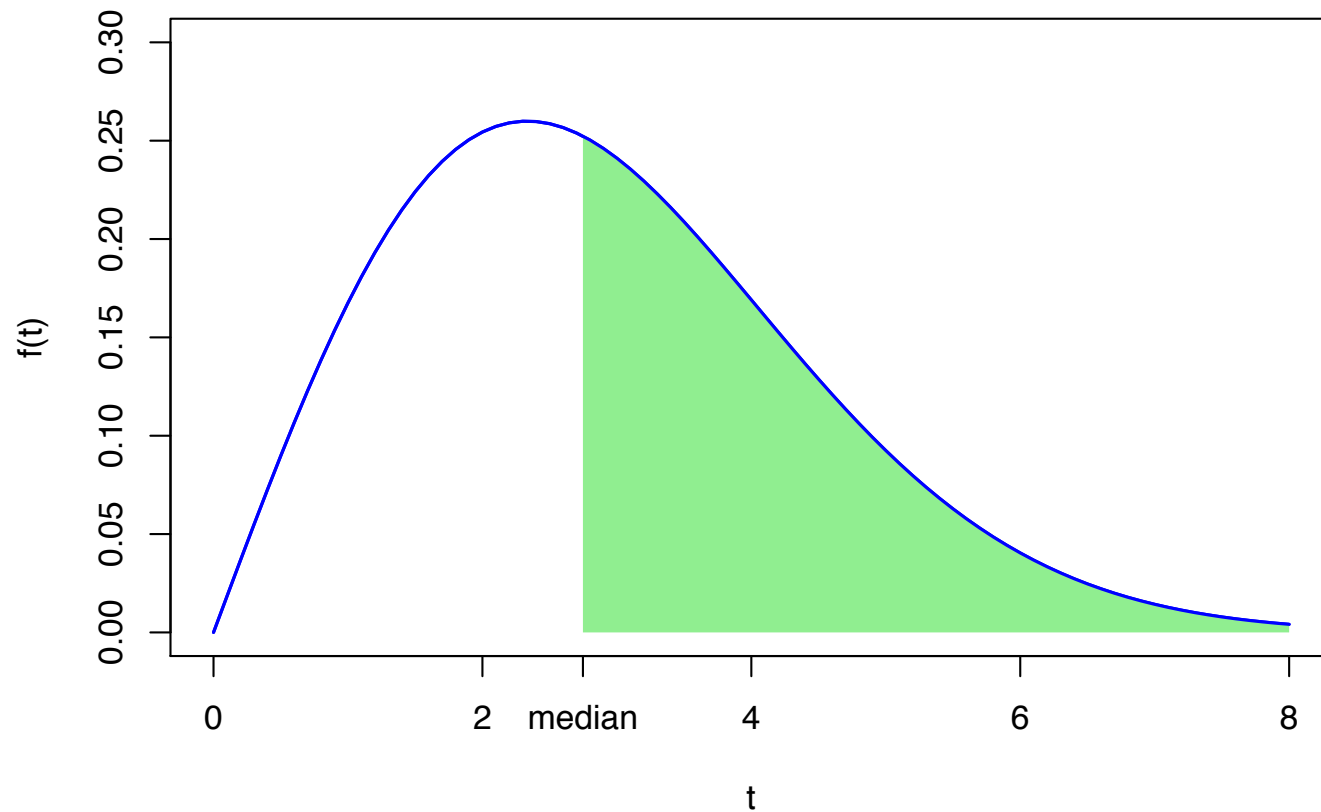


MEDIAN SURVIVAL TIME

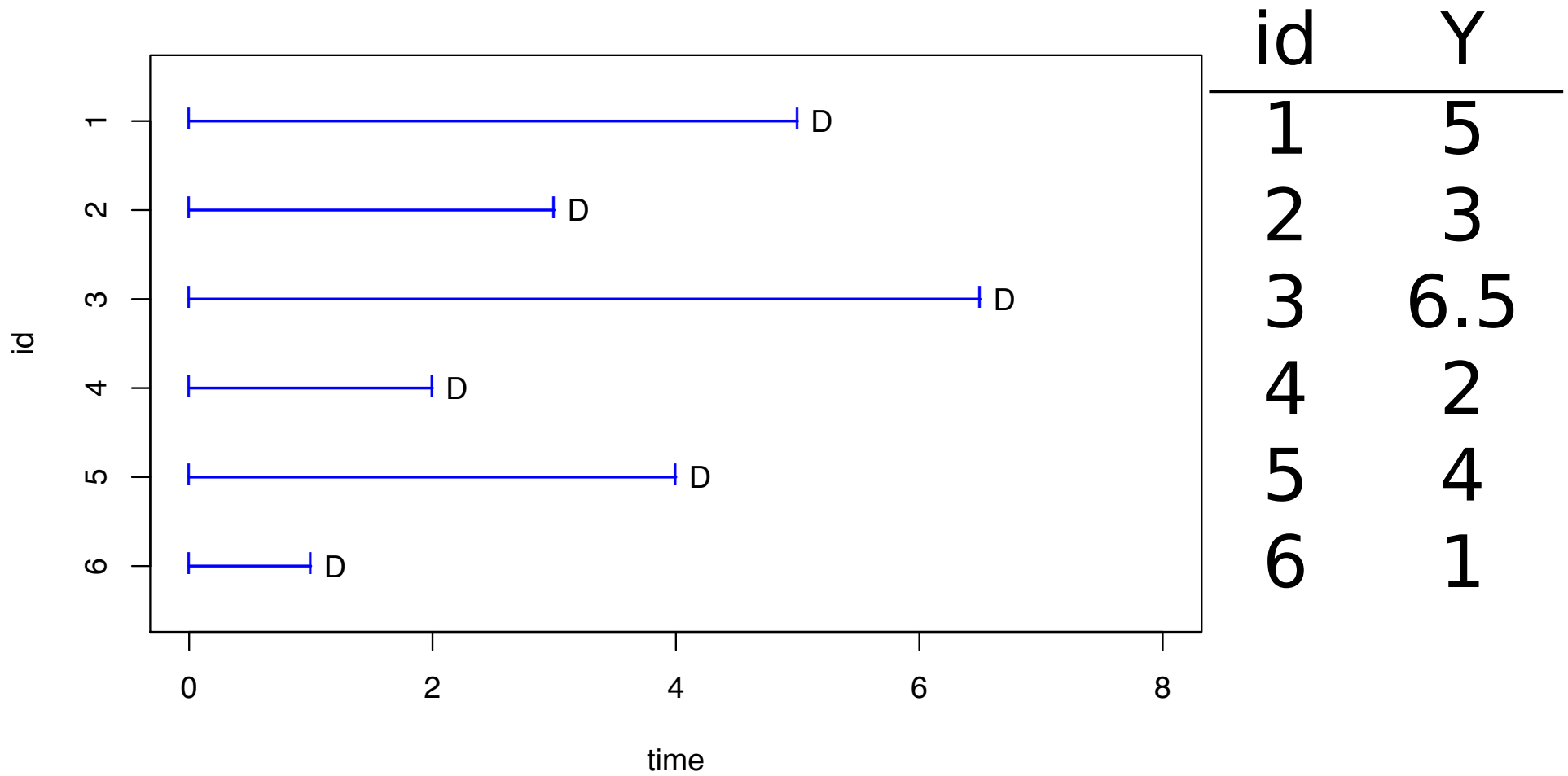


MEDIAN SURVIVAL TIME

Density Function

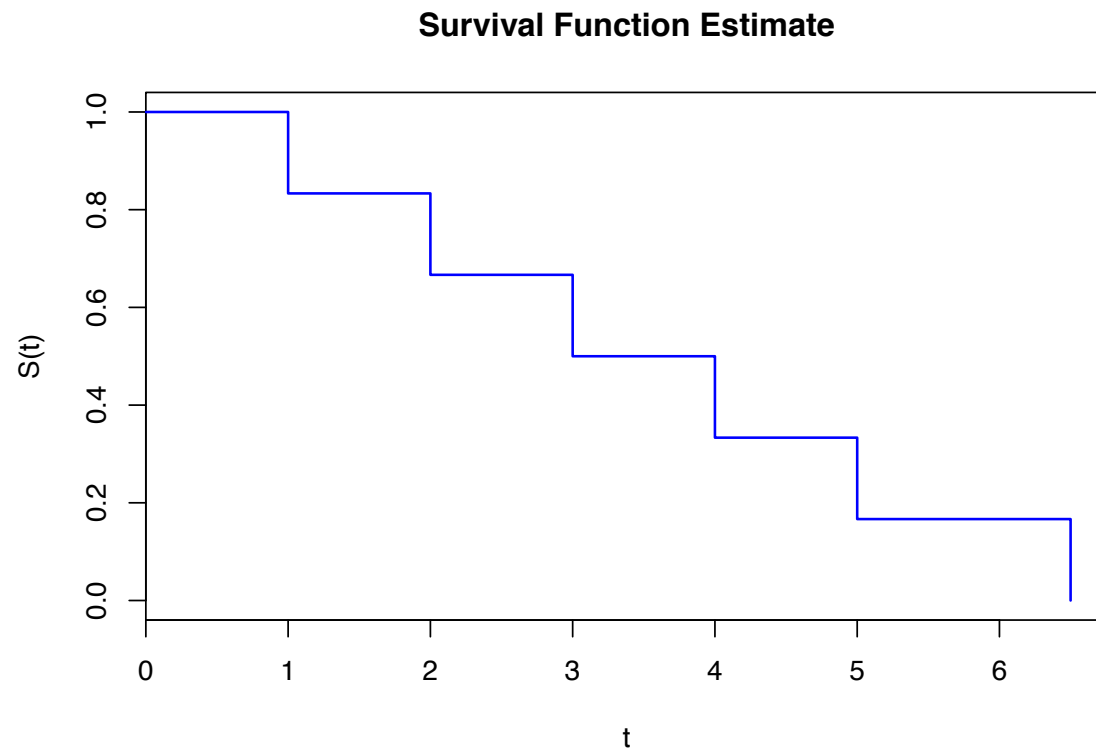


ILLUSTRATIVE DATA

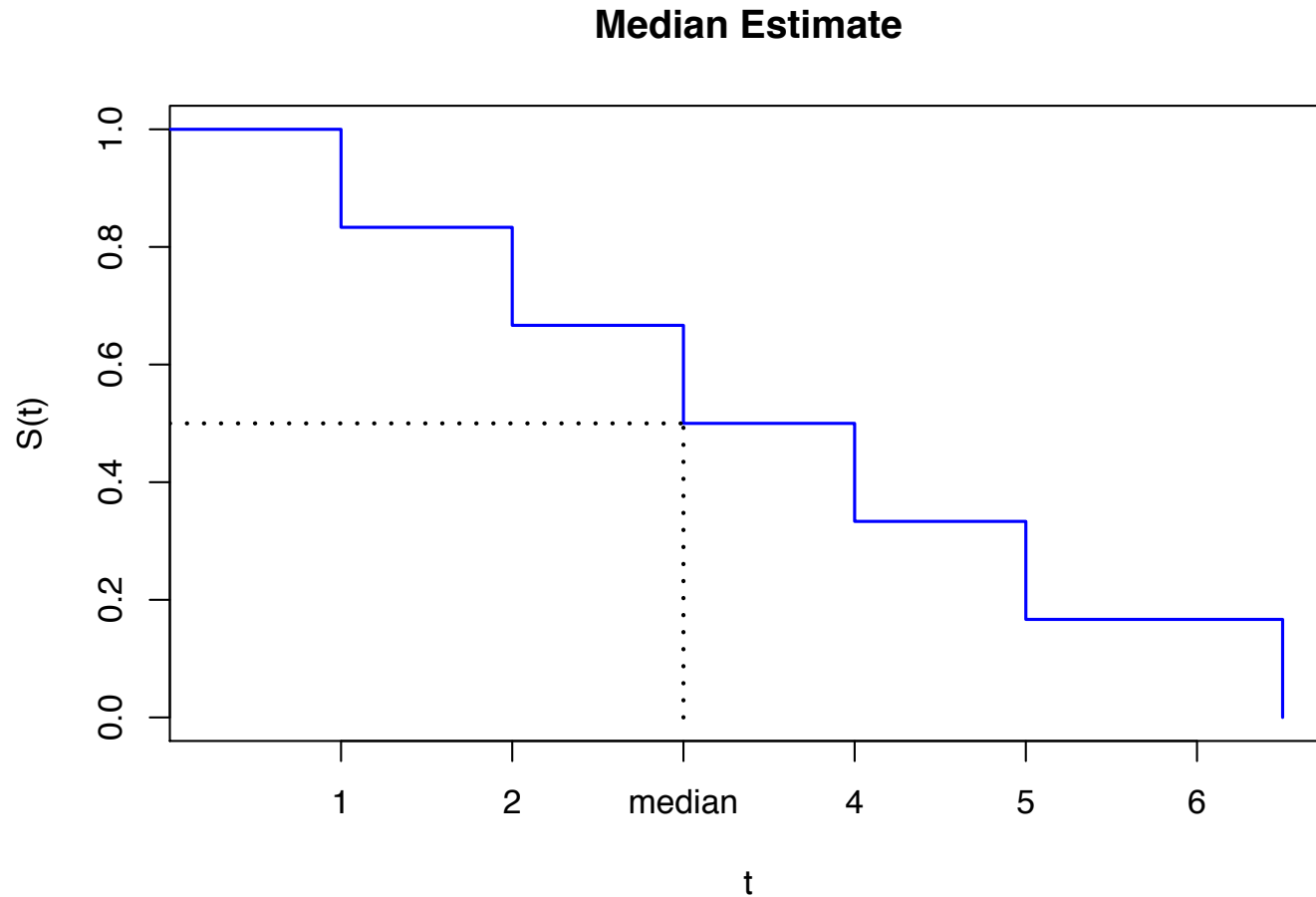


SURVIVAL FUNCTION ESTIMATE

- Nonparametric Estimate: reduce estimate by $1/n$ every time there is an event (death): Empirical survival function estimate



MEDIAN ESTIMATE



By convention: median is earliest time where survival estimate $\leq .5$

OTHER WAYS TO DESCRIBE A SURVIVAL DISTRIBUTION

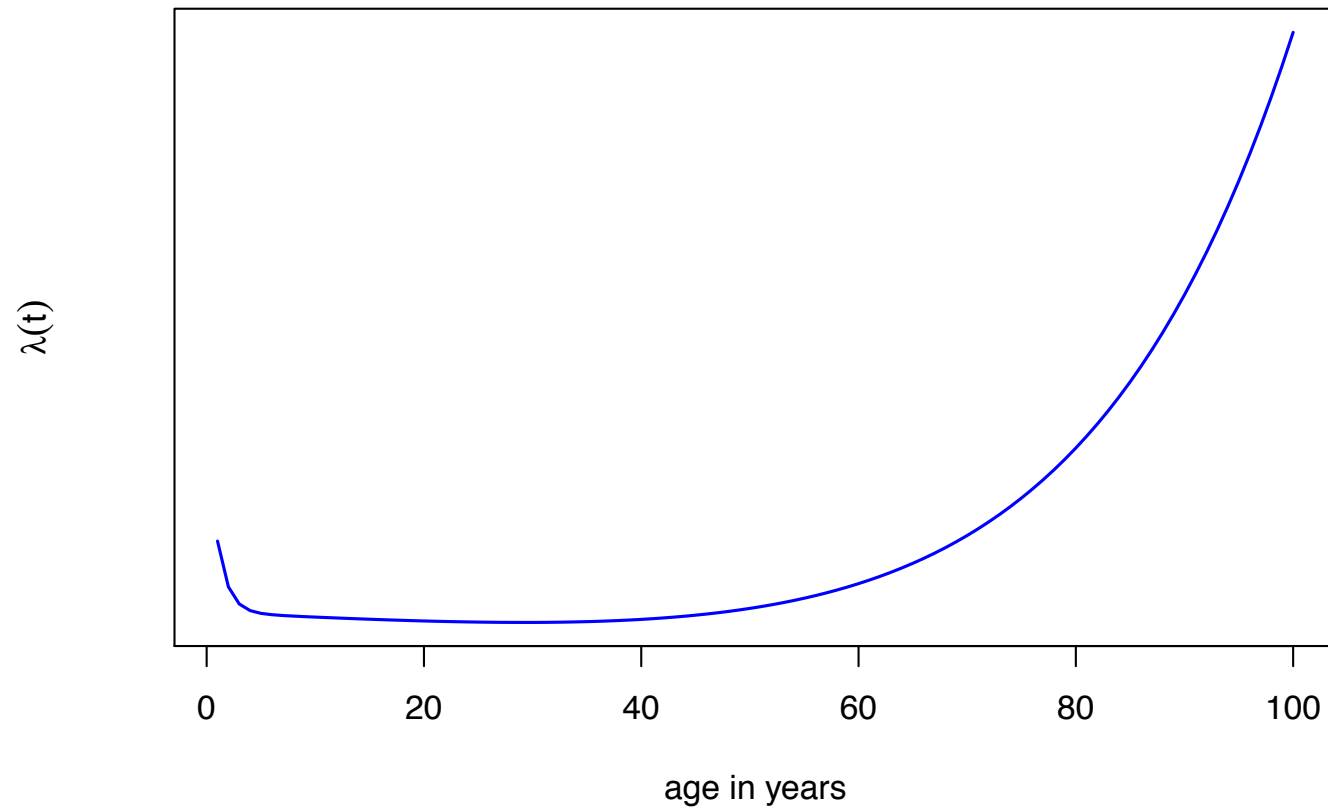
- So far we have looked at the density function and survival function $S(t)$.
- Also of interest: “hazard” function $\lambda(t)$

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \Pr[t \leq T < t + \Delta t | T \geq t]$$

- Instantaneous **rate** at which death occurs at t in those who are alive at t
- Examples:
 - Age-specific death rate
 - Age-specific disease incidence rate

HAZARD FUNCTION FOR HUMANS

Human Mortality



EQUIVALENT CHARACTERIZATIONS

- Any one of the density function($f(t)$), the survival function($S(t)$) or the hazard function($\lambda(t)$) is enough to determine the survival distribution.
- They are each functions of each other:

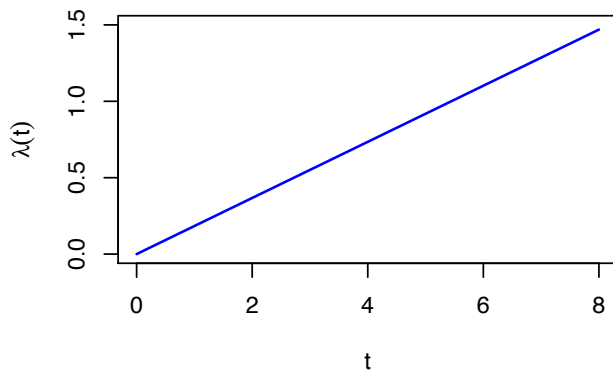
- $S(t) = \int_t^{\infty} f(s)ds = e^{-\int_0^t \lambda(s)ds}$

- $f(t) = -\frac{d}{dt}S(t) = \lambda(t)e^{-\int_0^t \lambda(s)ds}$

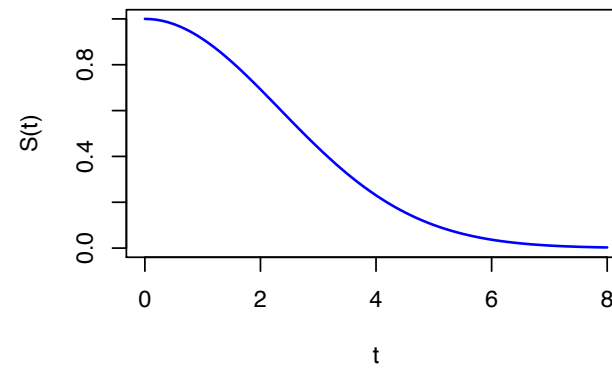
- $\lambda(t) = \frac{f(t)}{S(t)}$

EQUIVALENT CHARACTERIZATIONS

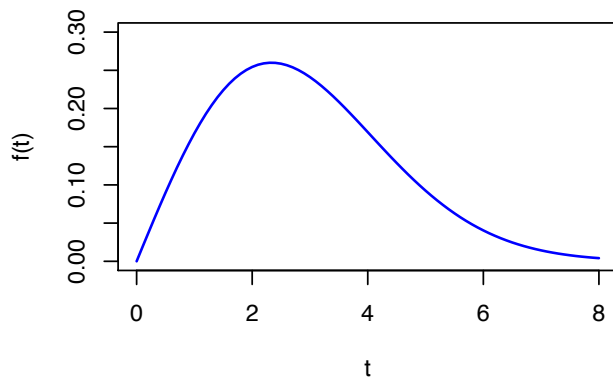
Hazard Function



Survival Function

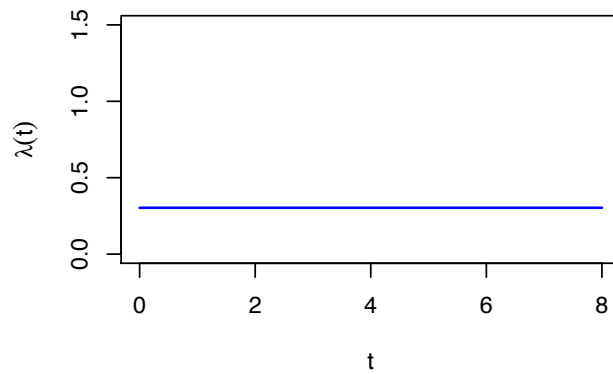


Density Function

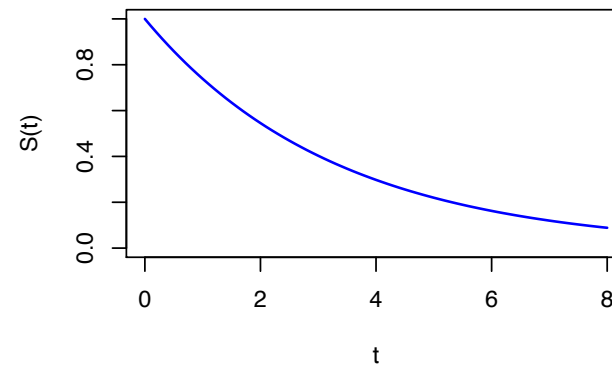


EQUIVALENT CHARACTERIZATIONS

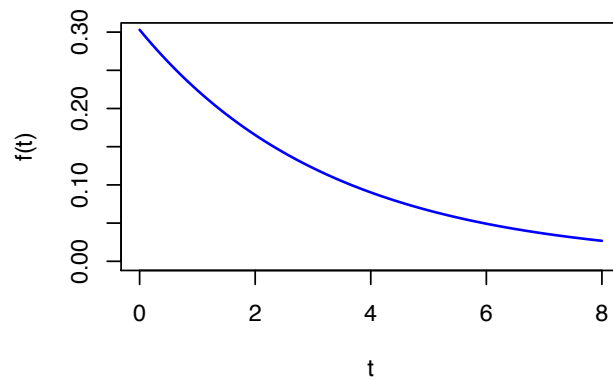
Hazard Function



Survival Function



Density Function



QUESTIONS?