

Statistical Learning in Mediation Analysis

Lab 1: Counterfactuals for mediation analysis

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Contents of this lab

- 1 Understanding nested counterfactuals
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Counterfactuals and nested counterfactuals

Assume we have a population consisting of only 5 individuals. The treatment and counterfactual values for mediator and outcome for these five individuals are given in the table below.

- Compute controlled direct effect $E[Y(1,0) - Y(0,0)]$
- Compute natural indirect effect $E[Y(1, M(1)) - Y(1, M(0))]$
- Compute the average treatment effect $E[Y(1) - Y(0)]$

Subject	A	M(1)	M(0)	Y(1,1)	Y(1,0)	Y(0,1)	Y(0,0)
1	1	1	0	1	1	1	0
2	0	0	0	1	0	1	0
3	0	0	1	1	0	0	0
4	1	1	1	0	0	0	1
5	0	1	1	0	1	0	1

Solution

Note that $Y(1, M(0))$ is equal to $Y(1, 1)$ if $M(0) = 1$ and equal to $Y(1, 0)$ if $M(0) = 0$. Using these definitions we have

Subject	$Y(1, M(1))$	$Y(1, M(0))$	$Y(0, M(1))$	$Y(0, M(0))$
1	1	1	1	0
2	0	0	0	0
3	0	1	0	0
4	0	0	0	0
5	0	0	0	0

- The controlled direct effect is equal to $E[Y(1, 0) - Y(0, 0)] = 2/5 - 2/5 = 0$
- The natural indirect effect is equal to $E[Y(1, M(1)) - Y(1, M(0))] = 1/5 - 2/5 = -1/5$
- The average treatment effect is equal to $E[Y(1) - Y(0)] = E[Y(1, M(1)) - Y(0, M(0))] = 1/5$.