

# Statistical Learning in Mediation Analysis

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## Lab 1: Counterfactuals for mediation analysis

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### MODULE 13

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## Contents of this lab

- 1 Understanding nested counterfactuals
- 2 Using nested counterfactuals to compute mediational effects

## Counterfactuals and nested counterfactuals

Assume we have a population consisting of only 5 individuals. The treatment and counterfactual values for mediator and outcome for these five individuals are given in the table below.

- Compute controlled direct effect  $E[Y(1, 0) - Y(0, 0)]$
- Compute natural indirect effect  $E[Y(1, M(1)) - Y(1, M(0))]$
- Compute the average treatment effect  $E[Y(1) - Y(0)]$

Subject	A	M(1)	M(0)	Y(1, 1)	Y(1, 0)	Y(0, 1)	Y(0, 0)
1	1	1	0	1	1	1	0
2	0	0	0	1	0	1	0
3	0	0	1	1	0	0	0
4	1	1	1	0	0	0	1
5	0	1	1	0	1	0	1

## Solution

Note that  $Y(1, M(0))$  is equal to  $Y(1, 1)$  if  $M(0) = 1$  and equal to  $Y(1, 0)$  if  $M(0) = 0$ . Using these definitions we have

Subject	$Y(1, M(1))$	$Y(1, M(0))$	$Y(0, M(1))$	$Y(0, M(0))$
1	1	1	1	0
2	0	0	0	0
3	0	1	0	0
4	0	0	0	0
5	0	0	0	0

- The controlled direct effect is equal to  $E[Y(1, 0) - Y(0, 0)] = 2/5 - 2/5 = 0$
- The natural indirect effect is equal to  $E[Y(1, M(1)) - Y(1, M(0))] = 1/5 - 2/5 = -1/5$
- The average treatment effect is equal to  $E[Y(1) - Y(0)] = E[Y(1, M(1)) - Y(0, M(0))] = 1/5$ .